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Binding Performance and Power of Dense Linear Algebra Operations

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Motivation

- High performance computing:
    - Optimization of algorithms applied to solve complex problems
- Technological advance ⇒ improve performance:
    - Higher number of cores per socket (processor)
- Large number of processors and cores ⇒ High energy consumption
- Tools to analyze performance and power in order to detect code inefficiencies and reduce energy consumption

Outline

1. Introduction
2. Tools for performance and power tracing
    - Performance tracing framework
    - Power tracing framework
    - Example
3. Experimental results
    - Environment setup
    - LU factorization
    - Cholesky factorization
    - Reduction to tridiagonal form
    - Results
4. Conclusions

Introduction

- **Parallel scientific applications**
    - Examples for dense linear algebra: Cholesky, QR and LU factorizations
- **Tools for power and energy analysis**
    - Power profiling in combination with Extrae+Paraver tools

Parallel applications + Power profiling

$\Downarrow$

Environment to identify sources of power inefficiency

$\Downarrow$

Energy savings

Introduction

- **Parallel scientific applications**
  - Examples for dense linear algebra: Cholesky, QR and LU factorizations
- **Tools for power and energy analysis**
  - Power profiling in combination with Extrae+Paraver tools

Parallel applications + Power profiling

$\Downarrow$

Environment to identify sources of power inefficiency

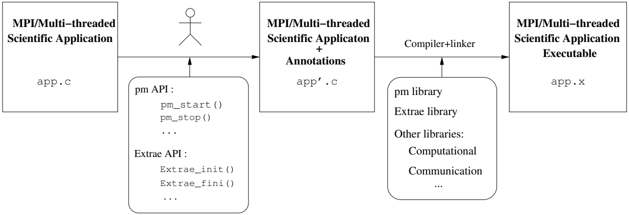
$\Downarrow$

Energy savings

Tools for performance and power tracing

Why traces?

- Details and variability are important (along time, processors, etc.)
- Extremely useful to analyze performance of applications, also at power level!



- Scientific application `app.c`
- Application with annotated code `app'.c`
- Executable code `app.x`

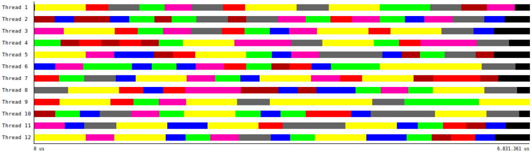
Tracing framework

`Extrae`: instrumentation and measurement package of BSC (Barcelona Supercomputing Center):

- Intercept calls to MPI, OpenMP, PThreads
- Records relevant information: time stamped events, hardware counter values, etc.
- Dumps all information into a single trace file.

`Paraver`: graphical interface tool from BSC to analyze/visualize trace files:

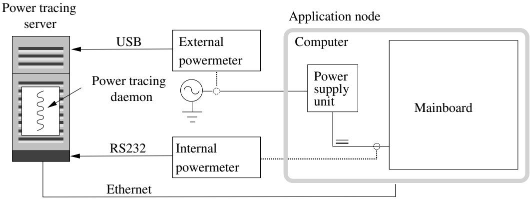
- Inspection of parallelism and scalability
- High number of metrics to characterize the program and performance application



Power measurement framework

`pmlib` library

- Power measurement package of Jaume I University (Spain)
- Interface to interact and utilize our own and commercial power meters



- **Server daemon**: collects data from power meters and send to clients
- **Client library**: enables communication with server and synchronizes with `start-stop` primitives

Power meter:

- ASIC-based powermeter (own design!)
- LEM HXS 20-NP transductors with PIC microcontroller
- Sampling rate 25 Hz

Scientific application

LU factorization with partial pivoting

$$PA = LU$$

$$\begin{aligned} A \in \mathbb{R}^{n\times n} &\quad \text{nonsingular matrix} \\ P \in \mathbb{R}^{n\times n} &\quad \text{permutation matrix} \\ L/U \in \mathbb{R}^{n\times n} &\quad \text{unit lower/upper triangular matrices} \end{aligned}$$

- Consider a partitioning of matrix A into blocks of size $b \times b$
- For numerical stability, permutations are introduced to prevent operation with small pivot elements

Example of performance and power tracing with the LU factorization:

- LAPACK routine `dgetrf`
- Shared-memory parallelism is extracted by calling to the multi-thread implementations of:
    - `dgetf2`, `dlaswp`, `dtrsm` and `dgemm` kernels from Intel MKL, AMD ACML or IBM ESSL.

Code annotation

LU factorization using LAPACK code:

```
#define Aref(i,j) A[((j)-1)*Alda+((i)-1)]
 void dgetrf( int m, int n, int b, double *A, int Alda, int *ipiv, int *info ){
  // Declaration of variables (omitted)


  for (j=1; j<=min( m, n ); j+=b) {

    // Factor current panel
    dgetf2( m-j+1, b, &Aref(j,j), Alda, &ipiv[j-1], info );



    // Apply permutations to left and right of panel
    dlaswp( j-1, A, Alda, j, j+b-1, ipiv, 1 );
    dlaswp( n-j-b+1, &Aref( 1, j+b ), Alda, j, j+b-1, ipiv, 1 );



    // Triangular solve
    dtrsm( "L", "L", "N", "U", b, n-j-b+1, done, &Aref( j, j ), Alda, &Aref( j, j+b ), Alda );



    // Update trailing submatrix
    dgemm( "N", "N", m-j-b+1, n-j-b+1, b, done, &Aref( j+b, j ), Alda,
           &Aref( j, j+b ), Alda,  done, &Aref( j+b, j+b ), Alda );

  }


}
```

Code annotation

LU factorization using LAPACK code (`Extrae` routines):

```
#define Aref(i,j) A[((j)-1)*Alda+((i)-1)]
void dgetrf( int m, int n, int b, double *A, int Alda, int *ipiv, int *info ){
  // Declaration of variables (omitted)

  Extrae_init();
  for (j=1; j<=min( m, n ); j+=b) {

    // Factor current panel
    dgetf2( m-j+1, b, &Aref(j,j), Alda, &ipiv[j-1], info );



    // Apply permutations to left and right of panel
    dlaswp( j-1, A, Alda, j, j+b-1, ipiv, 1 );
    dlaswp( n-j-b+1, &Aref( 1, j+b ), Alda, j, j+b-1, ipiv, 1 );



    // Triangular solve
    dtrsm( "L", "L", "N", "U", b, n-j-b+1, done, &Aref( j, j ), Alda, &Aref( j, j+b ), Alda );



    // Update trailing submatrix
    dgemm( "N", "N", m-j-b+1, n-j-b+1, b, done, &Aref( j+b, j ), Alda,
           &Aref( j, j+b ), Alda, done, &Aref( j+b, j+b ), Alda );

  }
  Extrae_fini();

}
```

Code annotation

LU factorization using LAPACK code (`Extrae` routines):

```
#define Aref(i,j) A[((j)-1)*Alda+((i)-1)]
void dgetrf( int m, int n, int b, double *A, int Alda, int *ipiv, int *info ){
  // Declaration of variables (omitted)

  Extrae_init();
  for (j=1; j<=min( m, n ); j+=b) {
    Extrae_event(500000001,1);
    // Factor current panel
    dgetf2( m-j+1, b, &Aref(j,j), Alda, &ipiv[j-1], info );
    Extrae_event(500000001,0);

    Extrae_event(500000001,2);
    // Apply permutations to left and right of panel
    dlaswp( j-1, A, Alda, j, j+b-1, ipiv, 1 );
    dlaswp( n-j-b+1, &Aref( 1, j+b ), Alda, j, j+b-1, ipiv, 1 );
    Extrae_event(500000001,0);

    Extrae_event(500000001,3);
    // Triangular solve
    dtrsm( "L", "L", "N", "U", b, n-j-b+1, done, &Aref( j, j ), Alda, &Aref( j, j+b ), Alda );
    Extrae_event(500000001,0);

    Extrae_event(500000001,4);
    // Update trailing submatrix
    dgemm( "N", "N", m-j-b+1, n-j-b+1, b, done, &Aref( j+b, j ), Alda,
           &Aref( j, j+b ), Alda,  done, &Aref( j+b, j+b ), Alda );
    Extrae_event(500000001,0);
  }
  Extrae_fini();

}
```

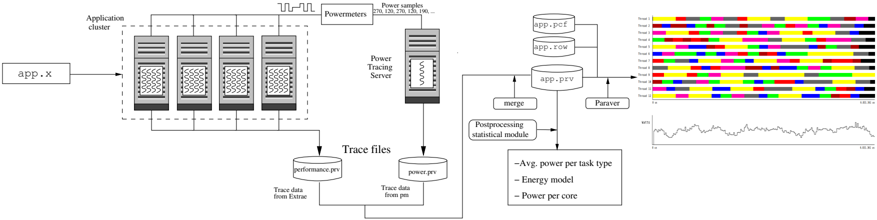
Code annotation

LU factorization using LAPACK code (`pmlib` routines):

```
#define Aref(i,j) A[((j)-1)*Alda+((i)-1)]
void dgetrf( int m, int n, int b, double *A, int Alda, int *ipiv, int *info ){
  // Declaration of variables (omitted)
  pm_start_counter(&pm_ctr);
  Extrae_init();
  for (j=1; j<=min( m, n ); j+=b) {
    Extrae_event(500000001,1);
    // Factor current panel
    dgetf2( m-j+1, b, &Aref(j,j), Alda, &ipiv[j-1], info );
    Extrae_event(500000001,0);

    Extrae_event(500000001,2);
    // Apply permutations to left and right of panel
    dlaswp( j-1, A, Alda, j, j+b-1, ipiv, 1 );
    dlaswp( n-j-b+1, &Aref( 1, j+b ), Alda, j, j+b-1, ipiv, 1 );
    Extrae_event(500000001,0);

    Extrae_event(500000001,3);
    // Triangular solve
    dtrsm( "L", "L", "N", "U", b, n-j-b+1, done, &Aref( j, j ), Alda, &Aref( j, j+b ), Alda );
    Extrae_event(500000001,0);

    Extrae_event(500000001,4);
    // Update trailing submatrix
    dgemm( "N", "N", m-j-b+1, n-j-b+1, b, done, &Aref( j+b, j ), Alda,
           &Aref( j, j+b ), Alda, done, &Aref( j+b, j+b ), Alda );
    Extrae_event(500000001,0);
  }
  Extrae_fini();
  pm_stop_counter(&pm_ctr);
}
```

Code execution

Basic execution schema for tracing performance and power:



Trace files:

- `Extrae` outputs `performance.prv` file
- `pmlib` outputs `power.prv` file

Tools:

- `Paraver`: performance and power trace visualization

Experimental results

Environment setup:

- 4 AMD Opteron 6172 processors, 4x12 cores at 2.1 GHz, 256 GB of RAM
- Intel MKL (v10.3.9) using IEEE double-precision arithmetic
- Performance traces obtained with `Extrae` (v2.2.0) and `Paraver` (v4.1.0)
- Power traces obtained with our power library `pmlib` (v2.0) and a microcontroller-based internal powermeter measuring 12 V motherboard lines at 25 samples/sec.
- Problem size: n=10,240

Implementations

LAPACK

- `Netlib` routines for:
    - LU factorization with partial pivoting (`dgetrf`)
    - Cholesky factorization (`dpotrf`)
    - Reduction to tridiagonal form (`dsytrd`)
- Parallelism exploited within the invocations to Intel (multi-threaded)
- 12 cores and block size *b*=128
- Routine `dpotrf` was modified to compute the Cholesky factorization via a right-looking algorithmic variant

MKL

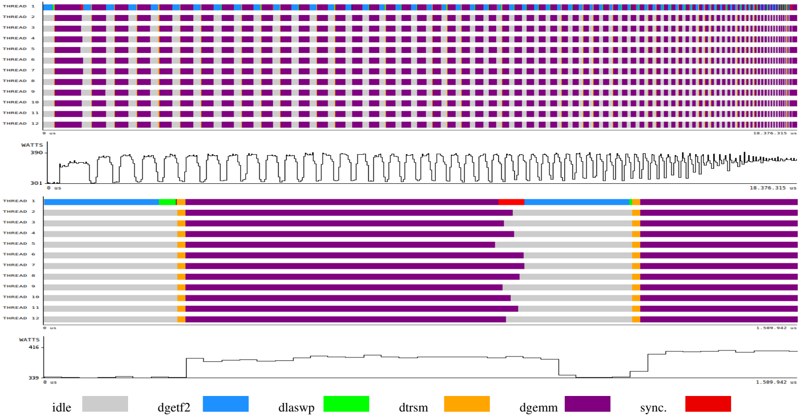
- Intel MKL routines for:
    - LU factorization with partial pivoting (`dgetrf`)
    - Cholesky factorization (`dpotrf`)
    - Reduction to tridiagonal form (`dsytrd`)
- 12 cores and block size *b*=128

SMPSs

- C codes for:
    - LU factorization with incremental pivoting
    - Cholesky factorization
- Linked to the sequential MKL BLAS, with task-level parallelism extracted by the SMPSs runtime system
- 6 cores, block size *b*=256 and internal block size *ib*=64

Experimental results: LU factorization

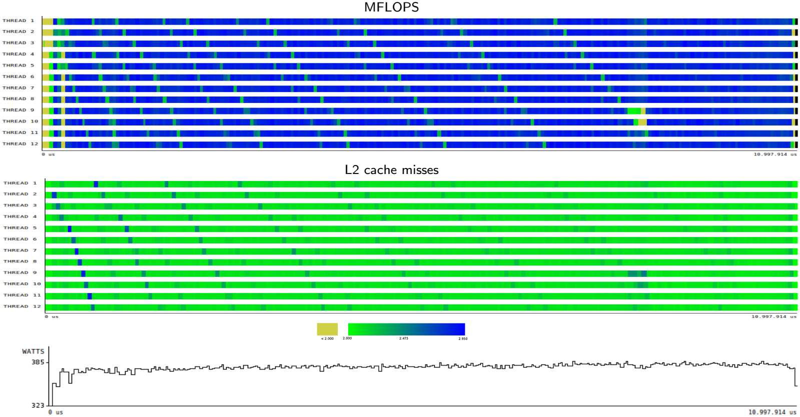
LU factorization with partial pivoting from LAPACK (`dgetrf`)



- Sequential execution of `dgetf2` and `dlaswp` (low power) and parallel execution for `dtrsm` and `dgemm` (high power)
- Synchronization points after `dgemm` execution, due to unbalanced distribution of work among cores

Experimental results: LU factorization

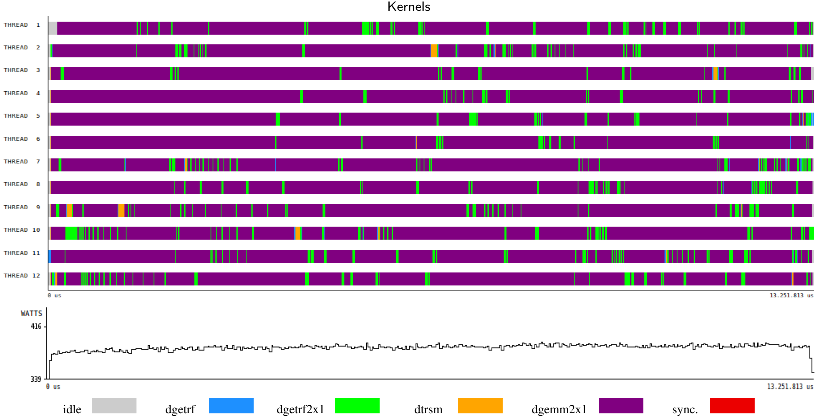
LU factorization with partial pivoting from MKL (`dgetrf`)



- `dgemm` and `dtrsm` are BLAS-3, thus deliver a high MFLOPS rate
- `dgetf2` is performed by only one core but overlapped with matrix updates (MKL code uses look-ahead techniques)
- Synchronization point at the end of execution ⇒ Algorithmic reasons

Experimental results: LU factorization

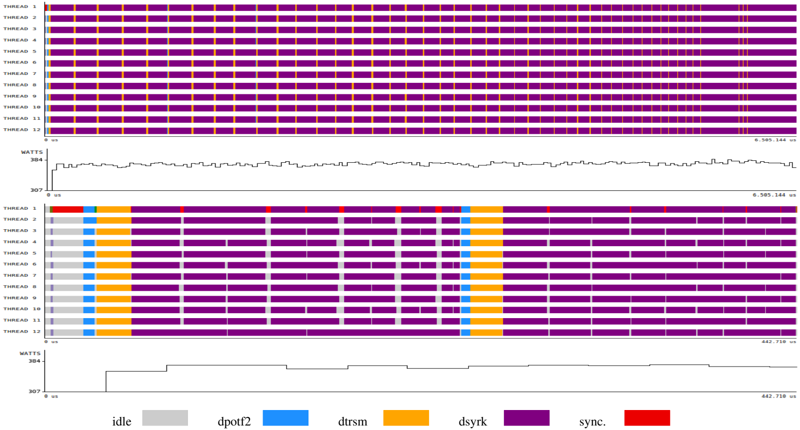
LU factorization with incremental pivoting parallelized with SMPS



- `dgemm2x1` dominates the execution time of the algorithm
- Plain power profile corresponding to `dgemm2x1` BLAS-3 kernel and the lack of idle periods

Experimental results: Cholesky factorization

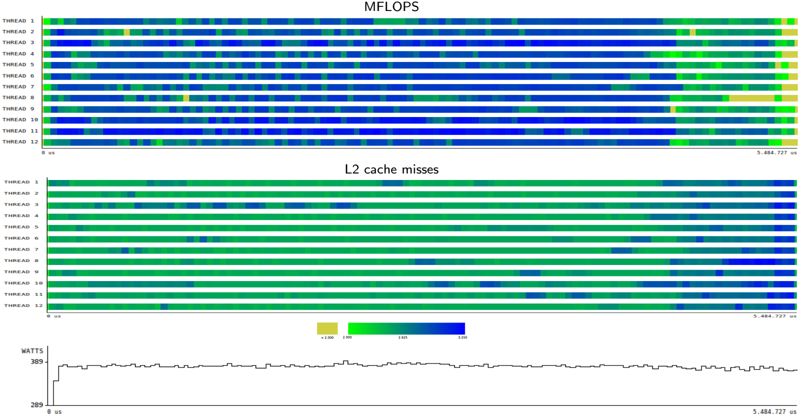
Cholesky factorization from LAPACK (`dpotrf`)



- Synchronization points due to unbalanced distribution of work among cores during `dsyrk` kernel ⇒ Idle periods
- Idle periods are so short and do not exert a visible change in the power profile

Experimental results: Cholesky factorization

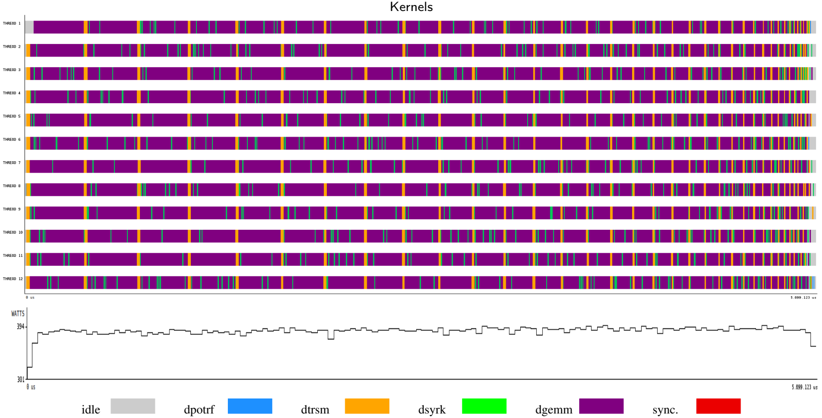
Cholesky factorization from MKL (`dpotrf`)



- High variability in MFLOPS rate taking into account that most of the operations are BLAS-3
- About 3/4 of the execution time a drastic decrease of MFLOPS is done ⇒ Change in MKL algorithm strategy
- Plain power profile even decreasing MFLOPS rate

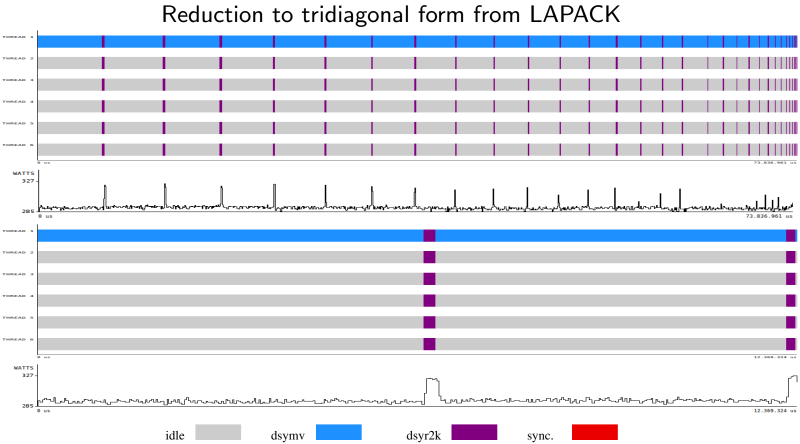
Experimental results: Cholesky factorization

Cholesky factorization parallelized with SMPS



- Better performance and low energy consumption of the SMPSs parallelization compared with the LAPACK and MKL implementations

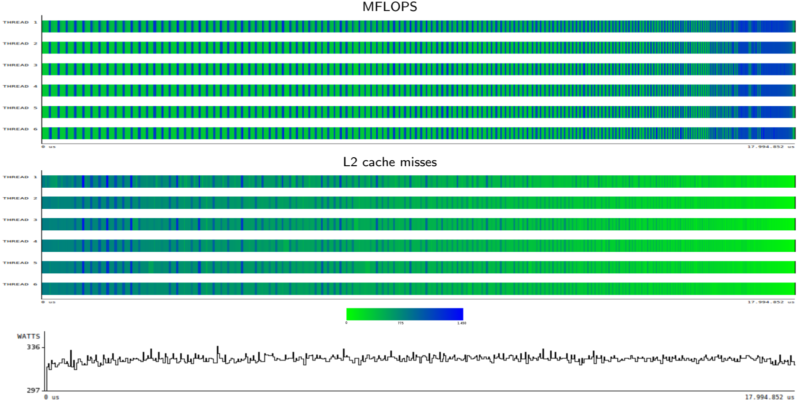
Experimental results: Reduction to tridiagonal form



- Interleaved execution of serial (`dsymv`) and parallel phases (`dsyr2k`)
- `dsymv` becomes a bottleneck because of the lack of concurrency of MKL implementation and low MFLOPS rate

Experimental results: Reduction to tridiagonal form

Reduction to tridiagonal form from MKL (`dsytrd`)



- Alternates periods of low and high activity for MFLOPS rate at high frequency!
- MKL employs a narrow block size to reduce latency of the panel factorization

Experimental results

Comparative table for evaluated algorithms and implementations:

	LU factorization			Cholesky factorization			Reduction to tridiagonal form	
	LAPACK	MKL	SMPSs	LAPACK	MKL	SMPSs	LAPACK	MKL
T (s)	18.37	10.99	13.25	6.50	5.48	5.09	73.83	17.99
GFLOPS	38.96	65.13	54.02	55.06	65.31	70.31	1.24	5.09
$P_{\max}$ (W)	390.70	385.78	392.81	384.61	389.06	393.52	327.42	336.33
$P_{\min}$ (W)	301.64	294.37	328.12	307.27	289.92	292.04	285.00	297.89
P_{avg} (W)	359.72	377.94	385.56	373.13	377.80	373.73	293.87	325.95
P_{wrk} (W)	112.22	130.44	138.06	125.63	130.30	125.23	46.37	78.45
E_{tot} (J)	6,608.60	4,155.61	5,109.44	2,427.28	2,072.07	1,905.70	21,698.53	5,865.51
E_{wrk} (J)	2,061.48	1,433.54	1,829.30	816.60	714.04	643.65	3,423.50	1,411.32

- *LU factorization*
  - Due to lack of synchronization points MKL leads better performance in terms of execution time over LAPACK
  - SMPSs: longer execution time due to high number of flops to perform LU factorization with incremental pivoting!
- *Cholesky factorization*
  - Superiority for the SMPSs parallelization from performance and energy!
  - SMPSs: Gains in execution time around 7% and improvement of energy savings about 9%
- *Reduction to tridiagonal form*
  - MKL outperforms the execution time of LAPACK due to a narrow block size and parallel version of `dsymv` kernel

Conclusions and future work

Implementations:

- MKL/SMPSs routines produce higher average power than LAPACK but provide a reduced execution time!
- MKL/SMPSs apply "race-to-idle" technique keeping the cores busy the most of the time!

MKL/SMPSs take advantage in energy efficiency!

Performance and power tracing:

- Detect code inefficiencies in order to reduce energy consumption
- Very useful to detect bottlenecks in the code:

Performance inefficiency ⇒ hot spots in hardware and power sinks in code

Future work:

- Developing power models for numerical libraries in order to predict energy consumption even without execution the code.

Thanks for your attention!

Questions?