

Fast Development of Dense Linear Algebra Codes on Graphics Processors

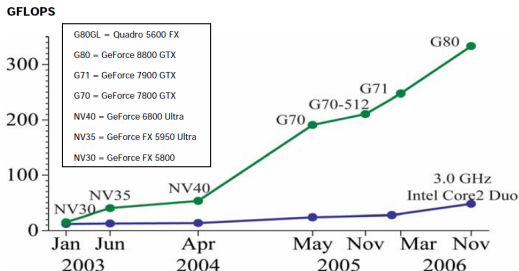
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High Performance Computing & Architectures Group
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Modern graphics processors have become a real and low cost alternative for HPC

Performance

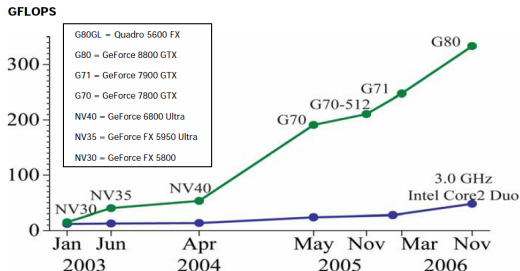


Easy to program

- Hardware:
 - More functionality
- Software:
 - Cg/OpenGL vs. CUDA
 - CUBLAS/CUFFT
 - ...

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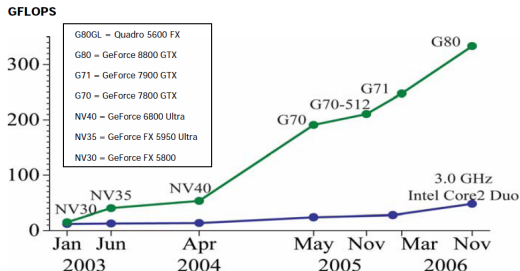


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Goals

- 1 Allow the development of robust codes
- 2 Reduce develop time
- 3 Reduce execution time

Our proposal: FLAG/C

Programming interface for dense linear algebra routines on GPUs

- **Robust codes:** FLAME methodology
- **Ease of use:** hides the underlying architecture
- **Performance:** GPU-based

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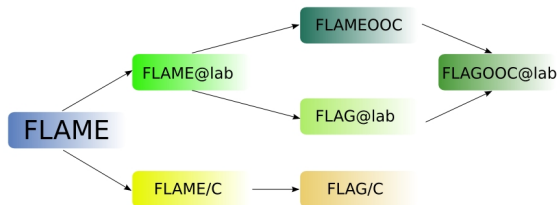
- 1 FLAG/C: interface description
- 2 Using the FLAME methodology on GPUs
- 3 Evaluation of the versatility, ease of use and efficiency of FLAG/C
- 4 Conclusion



The FLAG/C API

FLAG/C

- FLAG/C: high level API, written in C; fast development of efficient dense linear algebra codes.
- Can be combined with any accelerator type: GPUs, ClearSpeed boards, Cell B.E.
- Prerequisite: efficient implementation of BLAS available for the architecture

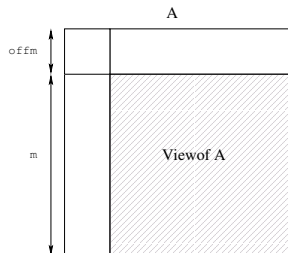




FLAG/C Objects

FLAG/C Objects

- Object: key concept in FLAG/C
- Representation of a matrix in the accelerator memory
- FLAG/C allows:
 - 1 Create, destroy and query the object properties
 - 2 Initialize the contents of an object
 - 3 Define *views*
 - 4 Operate with objects through BLAS



Views

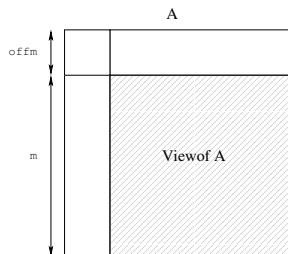
- Block references inside an object
- Flexibility to handle matrices and matrix blocks
- Without additional storage cost



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Views

- Block references inside an object
- Flexibility to handle matrices and matrix blocks
- Without additional storage cost



Typical structure of a FLAG/C program (I)

Initialization

```
FLA_Init() / FLA_Finalize()
```

Initialization / finalization of the environment



Object creation

```
FLAG_Obj_create( FLA_TYPE, int m, int n, FLA_Obj * )
```

Allocates space for the object in the GPU memory





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```
FLA_Init() / FLA_Finalize()
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Initialization / finalization of the environment



Object creation

```
FLAG_Obj_create( FLA_TYPE, int m, int n, FLA_Obj * )
```

Allocates space for the object in the GPU memory





Typical structure of a FLAG/C program (II)



CPU -> GPU Transfers

```
FLAG_Obj_set_with_buffer( void* buffer, FLA_Obj A )
```

Initializes the elements of object *A* in GPU memory with a copy of the contents of *buffer* in main memory



Matrix Computations

Complete BLAS implementation

```
FLAG_Gemm( FLAG_Trans transA, FLAG_Trans transB,  
FLA_Obj alpha, FLAG_Obj A, FLAG_Obj B, FLA_Obj beta,  
FLAG_Obj C )
```



Typical structure of a FLAG/C program (II)



CPU -> GPU Transfers

```
FLAG_Obj_set_with_buffer( void* buffer, FLA_Obj A )
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Initializes the elements of object *A* in GPU memory with a copy of the contents of *buffer* in main memory



Matrix Computations

Complete BLAS implementation

```
FLAG_Gemm( FLAG_Trans transA, FLAG_Trans transB,  
FLA_Obj alpha, FLAG_Obj A, FLAG_Obj B, FLA_Obj beta,  
FLAG_Obj C )
```



Typical FLAG/C program structure (III)



Utils

```
FLAG_Obj_show( char *s1, FLAG_Obj A, char * format,  
               char *s2 )
```

Shows the contents of object *A*



GPU -> CPU Transfers

```
FLAG_Obj_transfer_to_FLAG_Obj( FLAG_Obj A, FLAG_Obj B )
```

Initializes the elements of an object *B* with a copy of the contents of *A*, stored in GPU



Typical FLAG/C program structure (III)



Utils

```
FLAG_Obj_show( char *s1, FLAG_Obj A, char * format,  
               char *s2 )
```

Shows the contents of object *A*



GPU -> CPU Transfers

```
FLAG_Obj_transfer_to_FLAG_Obj( FLAG_Obj A, FLAG_Obj B )
```

Initializes the elements of an object *B* with a copy of the contents of *A*, stored in GPU



Example: $C = A^T \cdot B$

FLAG_Init ();

FLAG_Obj AObj, BObj, CObj;

FLAG_Obj_create('FLAG_FLOAT', n, n, &AObj);

FLAG_Obj_create('FLAG_FLOAT', n, n, &BObj);

FLAG_Obj_create('FLAG_FLOAT', n, n, &CObj);

FLAG_Obj_set_with_buffer((void *) A, &AObj);

FLAG_Obj_set_with_buffer((void *) B, &BObj);

FLAG_Obj_set_with_buffer((void *) C, &CObj);

FLAG_Gemm('FLAG_TRANSPOSE', ...
 'FLAG_NO_TRANSPOSE', ...
 1.0, &AObj, &BObj, 0.0, &CObj);

FLAG_Obj_show("C = [", CObj, "%f", "]");

FLAG_Obj_free(&AObj);

FLAG_Obj_free(&BObj);

FLAG_Obj_free(&CObj);

FLAG_Finalize ();

Initialization

Object creation

CPU -> GPU
Transfers

BLAS operations on
GPU

GPU memory free



Example: $C = A^T \cdot B$

```
FLAG_Init();
```

```
FLAG_Obj AObj, BObj, CObj;
```

```
FLAG_Obj_create( 'FLAG_FLOAT', n, n, &AObj );
```

```
FLAG_Obj_create( 'FLAG_FLOAT', n, n, &BObj );
```

```
FLAG_Obj_create( 'FLAG_FLOAT', n, n, &CObj );
```

```
FLAG_Obj_set_with_buffer( (void *) A, &AObj );
```

```
FLAG_Obj_set_with_buffer( (void *) B, &BObj );
```

```
FLAG_Obj_set_with_buffer( (void *) C, &CObj );
```

```
FLAG_Gemm( 'FLAG_TRANSPOSE', ...,
           'FLAG_NO_TRANSPOSE', ...,
           1.0, &AObj, &BObj, 0.0, &CObj );
```

```
FLAG_Obj_show( "C = [", CObj, "%f", "]" );
```

```
FLAG_Obj_free( &AObj );
```

```
FLAG_Obj_free( &BObj );
```

```
FLAG_Obj_free( &CObj );
```

```
FLAG_Finalize();
```

Initialization

Object creation

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FLAG_Init();
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FLAG_Obj_show( "C = [", CObj, "%f", "]" );
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```
FLAG_Obj_free( &AObj );
```

```
FLAG_Obj_free( &BObj );
```

```
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```
FLAG_Finalize();
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Object creation

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Example: $C = A^T \cdot B$

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FLAG_Init();
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```

```
FLAG_Obj_set_with_buffer( (void *) A, &AObj );
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```
FLAG_Obj_set_with_buffer( (void *) B, &BObj );
```

```
FLAG_Obj_set_with_buffer( (void *) C, &CObj );
```

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FLAG_Gemm( 'FLAG_TRANSPOSE', ...  
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          1.0, &AObj, &BObj, 0.0, &CObj );
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FLAG_Obj_show( "C = [", CObj, "%f", "]" );
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FLAG_Obj_free( &AObj );
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FLAG_Obj_free( &BObj );
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FLAG_Obj_free( &CObj );
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```
FLAG_Finalize();
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FLAG_Obj AObj, BObj, CObj;
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```
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```
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FLAG_Gemm( 'FLAG_TRANSPOSE', ...,
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FLAG_Obj_free( &AObj );
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FLAG_Obj_free( &BObj );
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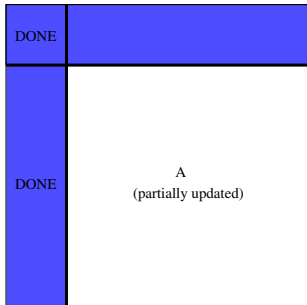


FLAME

- FLAME: fast development of robust and efficient libraries with a high abstraction level
- Focuses on dense and banded linear algebra
- High level of abstraction: hides implementation details
- It is not just a library:
 - Notation to express dense linear algebra algorithms
 - Methodology for systematic algorithm derivation
 - Programming interfaces (APIs) to represent algorithms in code almost effortlessly
 - Tools to ease the algorithmic derivation process
- Example: Cholesky factorization



Cholesky Factorization with FLAME



Definition

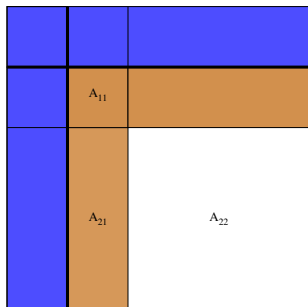
Given $A \rightarrow n \times n$ s.p.d.

$$A = LL^T$$

with $L \rightarrow n \times n$ lower triangular



Cholesky factorization with FLAME



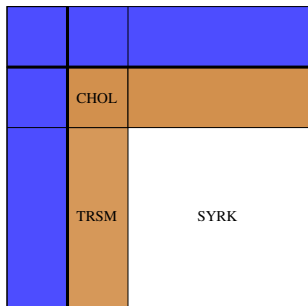
Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10}^T & A_{11} & A_{12}^T \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

where A_{11} is $b \times b$



Cholesky Factorization with FLAME

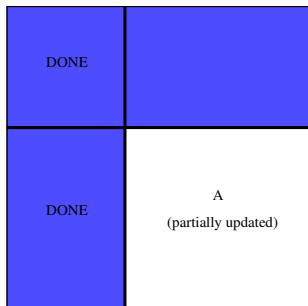


Operations per iteration:

```
%=====
A11 = chol( A11 );
A21 = A21 / tril( A11 )';
A22 = A22 - tril( A21 * A21' );
%=====
```



Cholesky Factorization with FLAME



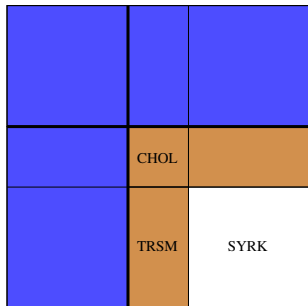
Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

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Cholesky Factorization with FLAME



Repartition

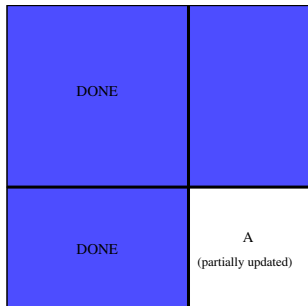
$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10}^T & A_{11} & A_{12}^T \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

where A_{11} is $b \times b$

```
FLA_Repart_2x2_to_3x3(
    ATL, ATR,          &A00, &A01, &A02,
                      &A10, &A11, &A12,
    ABL, ABR,          &A20, &A21, &A22,
                      b,  b, FLA_BR );
```



Cholesky Factorization with FLAME



Continue with

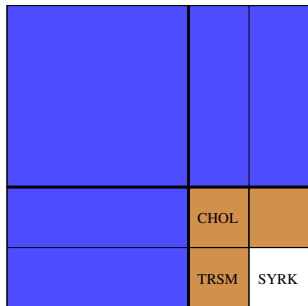
$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

where A_{11} is $b \times b$

```
FLA_Cont_with_3x3_to_2x2(
    &ATL, &ATR,      A00, A01, A02,
                      A10, A11, A12,
    &ABL, &ABR,      A20, A21, A22,
                      FLA_TL );
```



Cholesky Factorization with FLAME

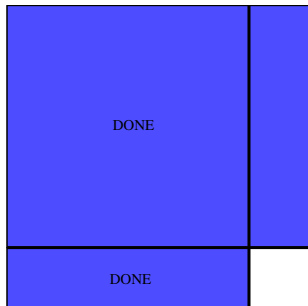


The algorithm iterates...

```
while ( size( ATL, 1 ) < size( A, 1 ) )  
...
```



Cholesky Factorization with FLAME



The algorithm iterates...

```
while ( size( ATL, 1 ) < size( A, 1 ) )  
...
```



FLAME and FLAME/C Notation

$$\text{Partition } A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$$

where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Determine block size n_b

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

where A_{11} is $n_b \times n_b$

$$A_{11} := \text{CHOL_UNB}(A_{11})$$

$$A_{21} := A_{21} \text{TRIL}(A_{11})^{-T}$$

$$A_{22} := A_{22} - A_{21}A_{11}^T$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

endwhile

```

int FLA_Chol_l_blk_var3_imp1( FLAG_Obj A, int nb_alg )
{
    FLA_Part_2x2( A,    &ATL, &ATR,
                  &ABL, &ABR,
                  0, 0, FLAG_TL );

    while ( FLA_Obj_width( ATL ) < FLA_Obj_width( A ) ){
        b = min( min( FLA_Obj_length( ABR ),
                     FLA_Obj_width( ABR ) ), nb_alg );

        FLA_Repart_2x2_to_3x3(
            ATL, ATR,      &A00, &A01, &A02,
                          &A10, &A11, &A12,
            ABL, ABR,      &A20, &A21, &A22,
                          b,  b, FLA_BR );

        /* ***** */
        FLA_Chol_unb( A11 );

        FLA_Trsm( FLA_RIGHT, FLA_LOWER_TRIANGULAR,
                  FLA_TRANSPOSE, FLA_NONUNIT_DIAG,
                  FLA_ONE, A11, A21 );

        FLA_Syrk( FLA_LOWER_TRIANGULAR, FLA_NO_TRANSPOSE,
                  FLA_MINUS_ONE, A21, FLA_ONE, A22 );

        /* ***** */
        FLA_Cont_with_3x3_to_2x2(
            &ATL, &ATR,      A00, A01, A02,
                          A10, A11, A12,
            &ABL, &ABR,      A20, A21, A22,
                          FLA_TL );
    }
}

```



Getting the FLAME/C code: Spark

Spark - Mozilla Firefox

Archivo Editar Ver Historial Marcadores Herramientas Ayuda

http://www.cs.utexas.edu/users/flame/Spark/

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Spark

Generate Code and/or Update Form Reset Form

[learn about this section](#) [introduction to Spark](#)

Name of the function to be generated
Cholesky

Type of function blocked

Variant Name 1

[learn about this section](#) [introduction to Spark](#)

Number of operands 1

Pick properties of the operands

Operand	Tag	Type	Direction	Input/Output
1:	A	matrix	TL->BR	input/output

[learn about this section](#) [introduction to Spark](#)

Pick an output language: FLAMEC

[learn about this section](#) [introduction to Spark](#)

Terminado

```
#include "FLAME.h"

int Cholesky_blk_var1( FLA_Obj A, int nb_alg )
{
    FLA_Obj ATL,   ATR,   A00, A01, A02,
             ABL,   ABR,   A10, A11, A12,
             A20, A21, A22;

    int b;

    FLA_Part_2x2( A,      6ATL, 6ATR,
                  6ABL, 6ABR,   0, 0, FLA_TL );

    while ( FLA_Obj_length( ATL ) < FLA_Obj_length( A ) ){
        b = min( FLA_Obj_length( ABR ), nb_alg );
        FLA_Repart_2x2_to_3x3( ATL, /**/ ATR,      6A00, /**/ 6A01, 6A02,
                               /* ***** */ /* ***** */
                               6A10, /**/ 6A11, 6A12,
                               ABL, /**/ ABR,      6A20, /**/ 6A21, 6A22,
                               b, b, FLA_BR );

        /*-----*/
        /*          update line 1          */
        /*          :                       */
        /*          update line n          */
        /*-----*/

        FLA_Cnt_with_3x3_to_2x2( 6ATL, /**/ 6ATR,      A00, A01, /**/ A02,
                                /* ***** */ /* ***** */
                                6ABL, /**/ 6ABR,      A10, A11, /**/ A12,
                                A20, A21, /**/ A22,
                                ... )
    }
```

The **Spark** tool eases the development process by automatically generating code skeletons



FLAME/C - FLAG/C Comparison

```

int FLAG_Chol_l_blk_var3_imp1( FLAG_Obj A, int nb_alg )
{
    FLAG_Part_2x2( A,    &ATL, &ATR,
                   &ABL, &ABR,
                   0, 0, FLAG_TL );

    while ( FLAG_Obj_width( ATL ) < FLAG_Obj_width( A ) ){
        b = min( min( FLAG_Obj_length( ABR ),
                     FLAG_Obj_width( ABR ) ), nb_alg);

        FLAG_Repart_2x2_to_3x3(
            ATL, ATR,          &A00, &A01, &A02,
                               &A10, &A11, &A12,
            ABL, ABR,          &A20, &A21, &A22,
                               b,  b, FLAG_BR );

        /* ***** */
        FLAG_Chol_unb( A11 );

        FLAG_Trsm( FLAG_RIGHT, FLAG_LOWER_TRIANGULAR,
                  FLAG_TRANSPOSE, FLAG_NONUNIT_DIAG,
                  FLA_ONE, A11, A21 );

        FLAG_Syrk( FLAG_LOWER_TRIANGULAR, FLAG_NO_TRANSPOSE,
                  FLA_MINUS_ONE, A21, FLA_ONE, A22 );

        /* ***** */

        FLAG_Cont_with_3x3_to_2x2(
            &ATL, &ATR,          A00, A01, A02,
                               A10, A11, A12,
            &ABL, &ABR,          A20, A21, A22,
                               FLA_TL );
    }
}

```

Main differences:

- 1 BLAS routines provided by FLAG/C
- 2 Matrices must have been transferred to the GPU



FLAG/C evaluation

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$
where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$$

where α_{11} is 1×1

Variant 1:

$$a_{10}^T := a_{10}^T \text{TRIL}(A_{00})^{-T}$$

$$\alpha_{11} := \alpha_{11} - a_{10}^T a_{10}$$

$$\alpha_{11} := \sqrt{\alpha_{11}}$$

Variant 2:

$$\alpha_{11} := \alpha_{11} - a_{10}^T a_{10}$$

$$\alpha_{11} := \sqrt{\alpha_{11}}$$

$$a_{21} := a_{21} - A_{20} a_{10}$$

$$a_{21} := a_{21} / \alpha_{11}$$

Variant 3:

$$\alpha_{11} := \sqrt{\alpha_{11}}$$

$$a_{21} := a_{21} / \alpha_{11}$$

$$A_{22} := A_{22} - a_{21} a_{21}^T$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right)$$

endwhile

Partition $A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right)$
where A_{TL} is 0×0

while $m(A_{TL}) < m(A)$ **do**

Determine block size n_b

Repartition

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

where A_{11} is $n_b \times n_b$

Variant 1:

$$A_{10} := A_{10} \text{TRIL}(A_{00})^{-T}$$

$$A_{11} := A_{11} - A_{10} A_{10}^T$$

$$A_{11} := \text{CHOL_UNB}(A_{11})$$

Variant 2:

$$A_{11} := A_{11} - A_{10} A_{10}^T$$

$$A_{11} := \text{CHOL_UNB}(A_{11})$$

$$A_{21} := A_{21} - A_{20} A_{10}^T$$

$$A_{21} := A_{21} \text{TRIL}(A_{11})^{-T}$$

Variant 3:

$$A_{11} := \text{CHOL_UNB}(A_{11})$$

$$A_{21} := A_{21} \text{TRIL}(A_{11})^{-T}$$

$$A_{22} := A_{22} - A_{21} A_{21}^T$$

Continue with

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & A_{01} & A_{02} \\ \hline A_{10} & A_{11} & A_{12} \\ \hline A_{20} & A_{21} & A_{22} \end{array} \right)$$

endwhile

- Square root calculations → CPU



FLAG/C evaluation

Implementation family **Imp1**

- Transfer of diagonal elements
 - Calculations
 - if $n_b \ll n \rightarrow$ lots of calls to level-1 and level-2 CUBLAS with low computational load
 - if n_b is large, the number of operations expressed in terms of level-3 CUBLAS is reduced
 - Communications
 - $T_1(n) = n(2\alpha + 8\beta)$ where $\alpha \gg \beta$
 - $2n$ transfers of 4 bytes each

Implementation family **Imp2** (Imp1 generalization)

- Transfer of diagonal blocks
- Calculations: level-3 CUBLAS calls
- Communications
 - $T_2(n, n_b) = \frac{n}{n_b} (2\alpha + 8\beta n_b^2)$
 - $2 \frac{n}{n_b}$ transfers of $4n_b^2$ bytes

FLAG/C evaluation

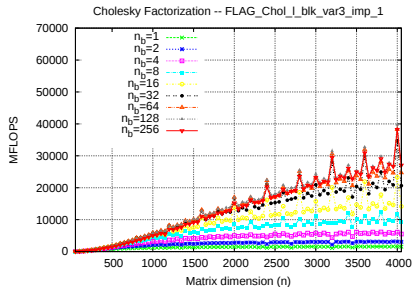
- Used metric: $\text{MFLOPs} = \frac{n^3}{3 \times 10^6 \times T_{ejec}}$
- T_{ejec} is the time used by the processor to calculate the factorization. It includes:
 - Transfer times of the matrix (CPU $\rightarrow$ GPU)
 - Transfer times of the Cholesky factor (GPU $\rightarrow$ CPU)

	CPU	GPU
Processor	Intel Core 2 Duo	NVIDIA 8800 Ultra
Model	Crusoe E6320	G80
Clock frequency	1.86 GHz	575 MHz
Memory frequency	2×333 MHz	2×900 MHz
Bus bandwidth	64 bits	384 bits
Max. bandwidth	5.3 GB/s	86.4 GB/s
Memory	1024 MB DDR2	768 MB GDDR3
Bus type	PCI Express x16 (4 GB/s)	
BLAS version	Intel MKL 10	CUBLAS 2.0

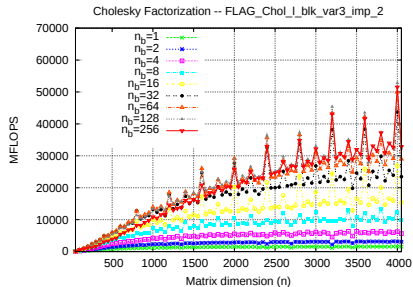


FLAG/C evaluation: Imp1 vs. Imp2

Imp1 Var3 CHOL_BLK



Imp2 Var3 CHOL_BLK

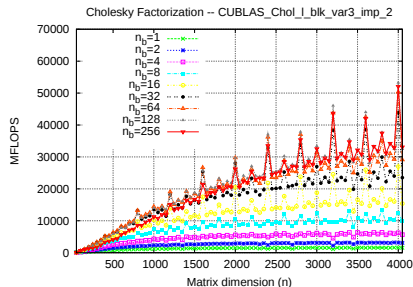


- Important improvement when $n \in [0, 2000]$
- Conclusions:
 - Versatility to get high performance implementations

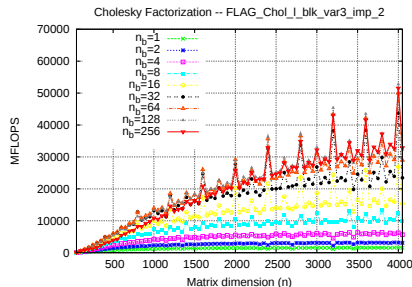


FLAG/C evaluation: overhead

CUBLAS
Imp2 Var3 CHOL_BLK



FLAG/C
Imp2 Var3 CHOL_BLK

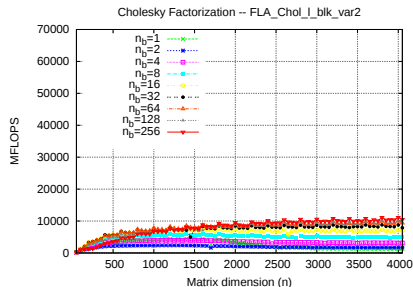


- The overhead introduced by FLAG/C is small

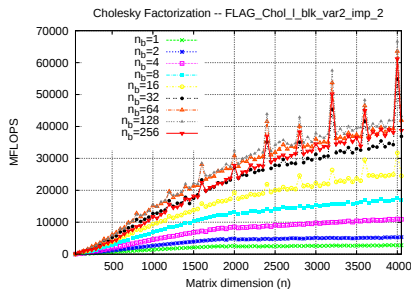


FLAG/C evaluation: FLAME/C vs. FLAG/C

FLAME/C Var2 CHOL_BLK



FLAG/C Imp2 Var2 CHOL_BLK



- The GPU can accelerate the calculation significantly (7x in our example)



The programmability problem (I)

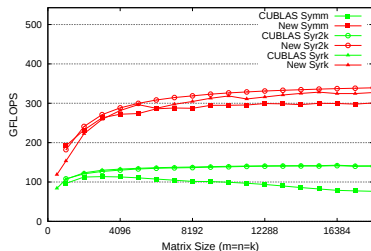
- Programmability: difficult to measure
- Success story: alternative level-3 (CU)BLAS implementation using FLAME
- GEMM in CUBLAS is the only fully tuned routine
- **Our goal:** develop different GEMM-based variants for each BLAS-3 routine in CUBLAS



The programmability problem (II)

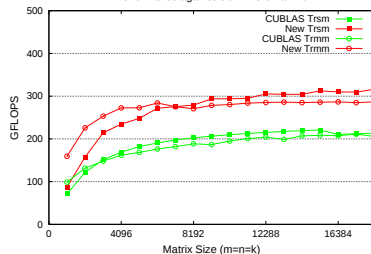
Our SSYRK, SSYMM, SSYR2K vs. CUBLAS

Performance against CUBLAS on a T10



Our STRSM, STRMM vs. CUBLAS

Performance against CUBLAS on a T10



- It took few hours to develop and test all the routines, algorithmic variants and block sizes.
- More information: FLAME Working Note #37.



Conclusion

- FLAG/C is **easy to use**:
 - The learning curve of FLAG/C is minimal
 - High abstraction level for the development of linear algebra codes
- FLAG/C is **efficient**:
 - Allows the exploitation of the massive multithread parallelism available on modern GPUs
 - Can attain up to 7x speedup compared with the CPU (for the shown Cholesky factorization)
- FLAG/C (FLAME) is **flexible**:
 - The interface can be adapted to any type of hardware accelerator



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Thank you...

More information. . .

<http://www3.uji.es/~figual>
<http://www.hpca.uji.es>
<http://www.cs.utexas.edu/~flame>