Level-3 BLAS on a GPU

Picking the Low Hanging Fruit

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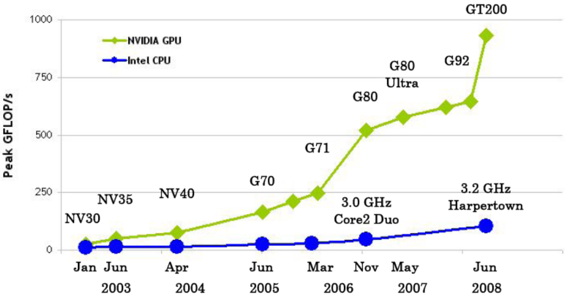
Motivation (I)

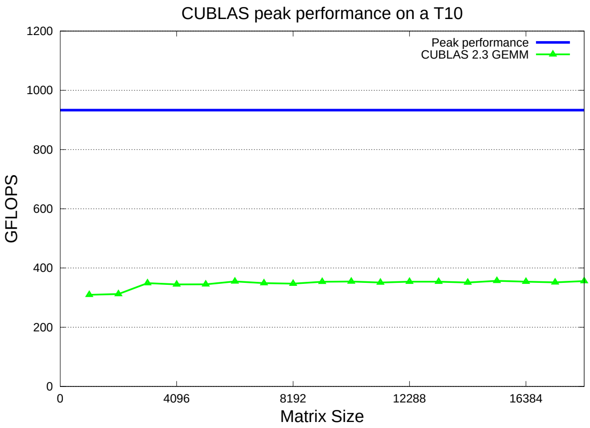
- GPU vendors promise spectacular peak performances



GT200 = GeForce GTX 280	G71 = GeForce 7900 GTX	NV35 = GeForce FX 5950 Ultra
G92 = GeForce 9800 GTX	G70 = GeForce 7800 GTX	NV30 = GeForce FX 5800
G80 = GeForce 8800 GTX	NV40 = GeForce 6800 Ultra	

Motivation (II)

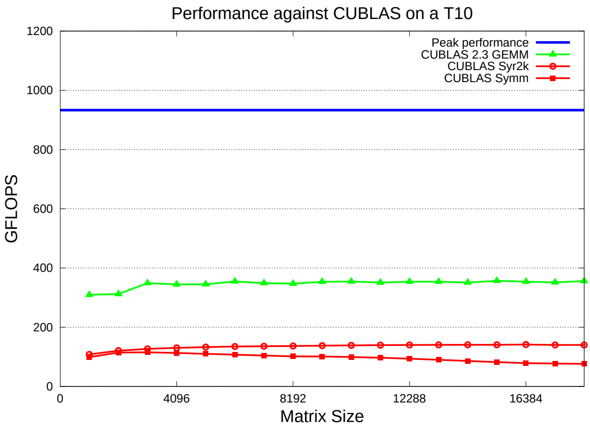
- But real performances are not so optimistic. . .



Limitations of the graphics-oriented architecture

Motivation (III)

- And current implementations can be quite poor. . .



Hard to efficiently program the GPU, even using CUDA

Outline

Contents

Introduction

BLAS

- BLAS: Basic Linear Algebra Subprograms
- Lie in the heart of complex dense linear algebra algorithms
- Key in their final performance
- Tuned implementations for many architectures
  - GotoBLAS, MKL, CUBLAS

Goals

- Tune the performance of the latest implementation of CUBLAS
- Without low-level programming (CUDA)
- Improving programmability: FLAME methodology

Introduction

BLAS

- BLAS: Basic Linear Algebra Subprograms
- Lie in the heart of complex dense linear algebra algorithms
- Key in their final performance
- Tuned implementations for many architectures
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Goals

- Tune the performance of the latest implementation of CUBLAS
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- Improving programmability: FLAME methodology

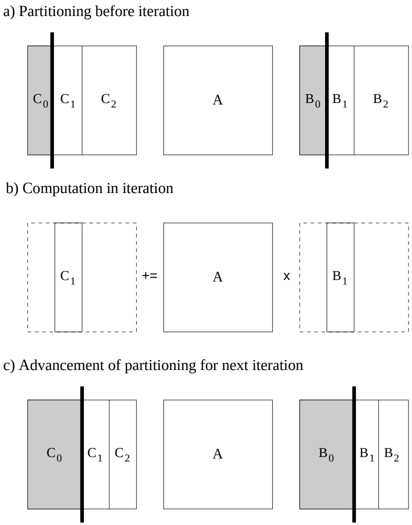
Contents

FLAME

The FLAME methodology

- FLAME: high level abstraction and notation for dense linear algebra algorithms
- Not only a library:
  - Notation for expressing algorithms
  - Methodology for systematic derivation of algorithms
  - Application Program Interfaces (APIs) for representing the algorithms in code
  - Tools and more
- Example: Matrix-matrix multiplication

The matrix-matrix multiplication



The matrix-matrix multiplication. FLAME algorithm

Algorithm: GEMM_MP(A,B,C)

Partition $B \rightarrow \left(B_L \,\middle|\, B_R \right)$, $C \rightarrow \left(C_L \,\middle|\, C_R \right)$
where B_L has 0 columns, C_L has 0 columns
while $n(B_L) < n(B)$ **do**
Determine block size b
Repartition

$$\left(B_L \,\middle|\, B_R \right) \rightarrow \left(B_0 \,\middle|\, B_1 \mid B_2 \right), \quad \left(C_L \,\middle|\, C_R \right) \rightarrow \left(C_0 \,\middle|\, C_1 \mid C_2 \right)$$

where B_1 has b columns, C_1 has b columns

$C_1 := C_1 + AB_1$

Continue with

$$\left(B_L \,\middle|\, B_R \right) \leftarrow \left(B_0 \mid B_1 \,\middle|\, B_2 \right), \quad \left(C_L \,\middle|\, C_R \right) \leftarrow \left(C_0 \mid C_1 \,\middle|\, C_2 \right)$$

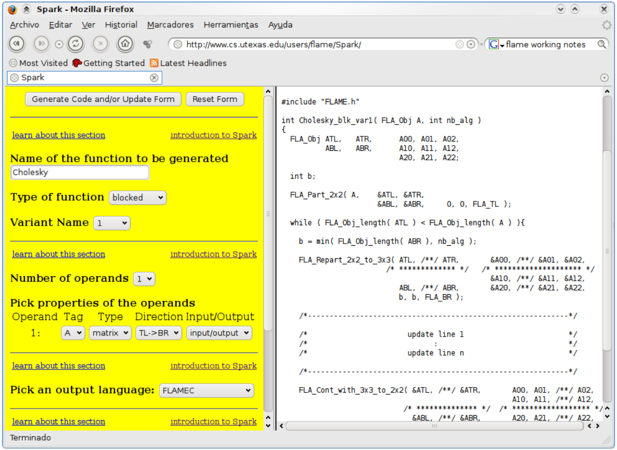
endwhile

The matrix-matrix multiplication. FLAME code

```
FLA_Obj   BL, BR,       B1, B2, B3;
FLA_Obj   CL, CR,       C1, C2, C3;

FLA_Part_1x2( B,
              &BL, &BR,              0, FLA_LEFT );

FLA_Part_1x2( C,
              &CL, &CR,              0, FLA_LEFT );

while ( FLA_Obj_width( BL ) < FLA_Obj_width( B ) ){

  b = min( FLA_Obj_width( BR ), nb_alg );

  FLA_Repart_1x2_to_1x3( BL,  /**/  BR,
                         &B0, /**/  &B1, &B2,
                                    b, FLA_RIGHT );

  FLA_Repart_1x2_to_1x3( CL,  /**/  CR,
                         &C0, /**/  &C1, &C2,
                                    b, FLA_RIGHT );

  /*----------------------------------------------------------------------*/
  FLA_Gemm( FLA_NO_TRANSPOSE, FLA_TRANSPOSE, FLA_MINUS_ONE, A, B1, FLA_ONE, C1 );
  /*----------------------------------------------------------------------*/

  FLA_Cont_with_1x3_to_1x2( &BL,     /**/  &BR,
                            B0, B1,  /**/  B2,
                                           FLA_LEFT );

  FLA_Cont_with_1x3_to_1x2( &CL,     /**/  &CR,
                            C0, C1,  /**/  C2,
                                           FLA_LEFT );
}
```

The matrix-matrix multiplication. Spark

SPARK: AUTOMATIC GENERATION OF CODE SKELETONS



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Accelerated Level-3 BLAS routines

SYMM $C := AB + C$, A symmetric and only the lower triangular part of this matrix is stored.

SYRK $C := C - AA^T$, C symmetric and only the lower triangular part of this matrix is stored and computed.

SYR2K $C := C - AB^T - BA^T$, C symmetric and only the upper triangular part of this matrix is stored and computed.

TRMM $C := AB + C$, where A upper triangular.

TRSM $XA^T = B$, A lower triangular and B is overwritten with the solution X.

Accelerating the CUBLAS

Three main ideas

1. GEMM-based implementations
2. Multiple algorithmic variants
3. Multiple block sizes

GEMM-based SYRK

Algorithm: SYRK_GEMM(C,A)

Partition $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$, $A \rightarrow \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right)$

where C_{TL} is 0×0, A_T has 0 rows

while $m(C_{TL}) < m(C)$ **do**

Determine block size b

Repartition

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & C_{01} & C_{02} \\ \hline C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline A_1 \\ A_2 \end{array}\right)$$

where C_{11} is $b \times b$, A_1 has b rows

$C_{11} := C_{11} - A_1 A_1^T$ $(SYRK)$

$C_{21} := C_{21} - A_2 A_1^T$ $(GEMM)$

Continue with

$$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ \hline C_{20} & C_{21} & C_{22} \end{array}\right), \left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ A_1 \\ \hline A_2 \end{array}\right)$$

endwhile

Multiple algorithmic variants

Algorithm: GEMM_MP(A, B, C)

Partition $B \rightarrow \left(B_L \,\middle|\, B_R \right)$, $C \rightarrow \left(C_L \,\middle|\, C_R \right)$
where B_L has 0 columns, C_L has 0 columns

while $n(B_L) < n(B)$ **do**

Determine block size b

Repartition

$$\left(B_L \,\middle|\, B_R \right) \rightarrow \left(B_0 \,\middle|\, B_1 \,|\, B_2 \right), \quad \left(C_L \,\middle|\, C_R \right) \rightarrow \left(C_0 \,\middle|\, C_1 \,|\, C_2 \right)$$

where B_1 has b columns, C_1 has b columns

$$C_1 := C_1 + AB_1$$

Continue with

$$\left(B_L \,\middle|\, B_R \right) \leftarrow \left(B_0 \,|\, B_1 \,\middle|\, B_2 \right), \quad \left(C_L \,\middle|\, C_R \right) \leftarrow \left(C_0 \,|\, C_1 \,\middle|\, C_2 \right)$$

endwhile

Multiple algorithmic variants

Algorithm: GEMM_PM(A,B,C)

Partition $A \rightarrow \left(\frac{A_T}{A_B}\right)$, $C \rightarrow \left(\frac{C_T}{C_B}\right)$

where A_T has 0 rows, C_T has 0 rows

while $m(A_T) < m(A)$ **do**

Determine block size b

Repartition

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \rightarrow \left(\begin{array}{c} A_0 \\ \hline A_1 \\ A_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \rightarrow \left(\begin{array}{c} C_0 \\ \hline C_1 \\ C_2 \end{array}\right)$$

where A_1 has b rows, C_1 has b rows

$$C_1 := C_1 + A_1 B$$

Continue with

$$\left(\begin{array}{c} A_T \\ \hline A_B \end{array}\right) \leftarrow \left(\begin{array}{c} A_0 \\ A_1 \\ \hline A_2 \end{array}\right), \left(\begin{array}{c} C_T \\ \hline C_B \end{array}\right) \leftarrow \left(\begin{array}{c} C_0 \\ C_1 \\ \hline C_2 \end{array}\right)$$

endwhile

Multiple algorithmic variants

Algorithm: GEMM_PP(A,B,C)

Partition $A \rightarrow \left(A_L \,\big|\, A_R \right)$, $B \rightarrow \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right)$

where A_L has 0 columns, B_T has 0 rows

while $n(A_L) < n(A)$ **do**

Determine block size b

Repartition

$$\left(A_L \,\big|\, A_R \right) \rightarrow \left(A_0 \,\big|\, A_1 \,|\, A_2 \right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$$

where A_1 has b columns, B_1 has b rows

$$C := C + A_1 B_1$$

Continue with

$$\left(A_L \,\big|\, A_R \right) \leftarrow \left(A_0 \,|\, A_1 \,\big|\, A_2 \right), \left(\begin{array}{c} B_T \\ \hline B_B \end{array} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$$

endwhile

Varying the block size

Algorithm: GEMM_PP(A,B,C)

Partition $A \rightarrow \left(A_L \,\middle|\, A_R \right)$, $B \rightarrow \left(\frac{B_T}{B_B} \right)$

where A_L has 0 columns, B_T has 0 rows

while $n(A_L) < n(A)$ **do**

Determine block size b

Repartition

$$\left(A_L \,\middle|\, A_R \right) \rightarrow \left(A_0 \,\middle|\, A_1 \mid A_2 \right), \left(\frac{B_T}{B_B} \right) \rightarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$$

where A_1 has b columns, B_1 has b rows

$$C := C + A_1 B_1$$

Continue with

$$\left(A_L \,\middle|\, A_R \right) \leftarrow \left(A_0 \mid A_1 \,\middle|\, A_2 \right), \left(\frac{B_T}{B_B} \right) \leftarrow \left(\begin{array}{c} B_0 \\ \hline B_1 \\ \hline B_2 \end{array} \right)$$

endwhile

Contents

Experimental setup

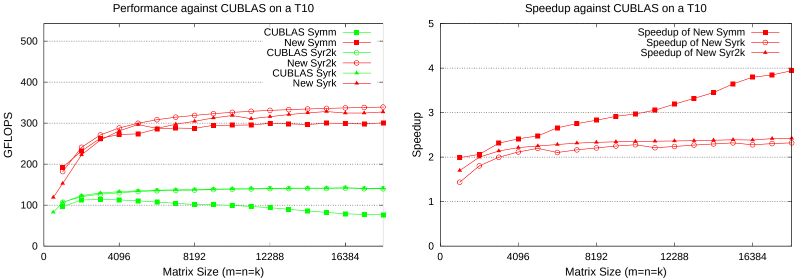
Experimental setup

CPU	Dual Xeon QuadCore E5410
CPU frequency	2.33 Ghz
RAM memory	8 Gbytes
GPU	Tesla C1060
Processor	Nvidia GT200
GPU frequency	1.3 Ghz
Video memory	4 Gbytes DDR3
Interconnection	PCIExpress Gen2
CUDA (CUBLAS) version	2.3 (July 2009)
Driver version	185.18

- Results in terms of GFLOPS (single precision)
- Transfer times not included in results

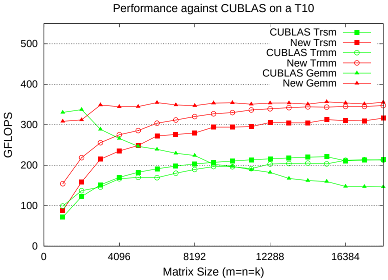
Experimental results

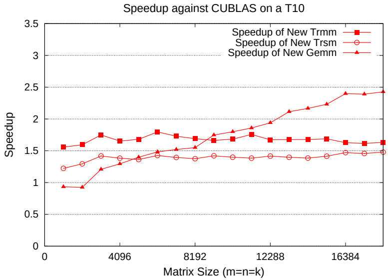
Results for square matrices: SYRK, SYMM and SYR2K.



Experimental results

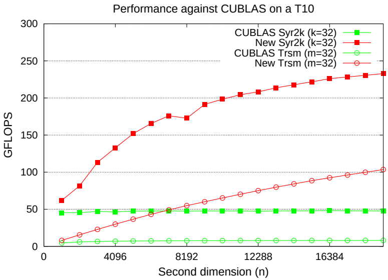
Results for square matrices: TRSM and TRMM.

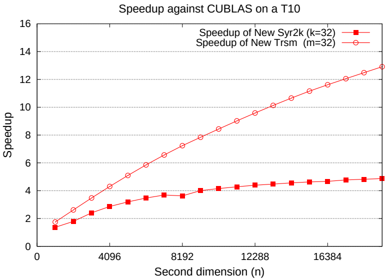




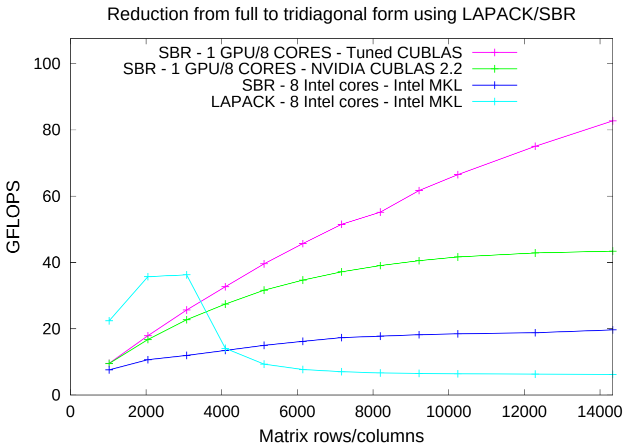
Experimental results

Results for rectangular matrices: TRSM and SYR2K.





Application: Symmetric Eigenvalue Problem (PPAM09)



- Using our SYMM and SYR2K implementations
- Speedup $2.2x$ when using GPU acceleration for SBR (19.2 vs 42 GFLOPS)
- Speedup $4.3x$ when using GPU acceleration and tuned BLAS (19.2 vs 82 GFLOPS)

Contents

Conclusions and future work

- Performance boost with little effort for BLAS routines
- Attain considerable speedups compared with tuned CUBLAS
- Without writing one line of CUDA code
- Methodology appliable to other linear algebra routines

Conclusions and future work

Thank you!

More information. . .

[Level-3 BLAS on a GPU: Picking the Low Hanging Fruit]
FLAME Working Note #37
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