

Energy-Aware Matrix Computations on Multi-Core and Many-core Platforms

Enrique S. Quintana-Ortí





Performance and energy consumption



Top500 (November 2011)

Rank	Site	#Cores	LINPACK (TFLOPS)
1	RIKEN AICS K Computer- Spar64 VIIIfx (8-core)	705,024	10,510.00*
2	NSC Tianjin – NUDT YH MPP, Xeon X5670 6C 2.93 GHz, NVIDIA 2050	186,368	2,566.00
3	DOE ORNL – Cray XT5-HE Opteron 6-core 2.6 GHz	224,162	1,759.00
9	CEA (France) – Bull bullx super-node S6010/S6030	138,368	1,050.00
114	BSC (Spain) – Bull B505, Xeon E5649 6C 2.53 GHz, NVIDIA 2090	5,544	103.20

¹ day K Computer = 394 years of the world population (7.000 million people) with a hand calculator

NIVERSITAT

Performance and energy consumption



Green500 (November 2011)

Rank	Site	#Cores	MFLOPS/W	LINPACK (TFLOPS)
Green/Top				(25. 3)
1/29	IBM Rochester – BlueGene/Q, Power BQC 16C 1.60 GHz	32,768	2,026.48	339.83
7/114	BSC (Spain) – Bull B505, Xeon E5649 6C 2.53 GHz, NVIDIA 2090	5,544	1,266.26	103.20
32/1	RIKEN AICS K Computer– Spar64 VIIIfx (8-core)	705,024	830.18	10,510.00
47/2	NSC Tianjin – NUDT YH MPP, Xeon X5670 6C 2.93 GHz, NVIDIA 2050	186,368	635.15	2,566.00
53/3	DOE ORNL – Cray XT5-HE Opteron 6-core 2.6 GHz	582,00	Cray	1,759.00



Multi-core and many-core platforms



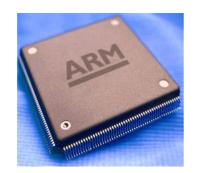
"Conventional" architectures







New challengers...







Matrix computations



- Linear algebra? Please, don't run away!
 - Determinants, linear systems, least squares fitting, FFT, etc.



- Importance:
 - Intel MKL, AMD ACML, IBM ESSL, NVIDIA CUBLAS, ongoing for TI



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- 1. Scientific applications
- 2. Leveraging concurrency
- 3. Cost of energy



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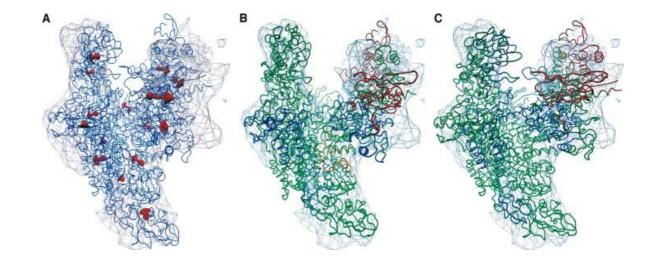


Scientific applications Biological systems



- Simulations of molecular dynamics
- Solve

$$AX = BX\Lambda$$
,
dense $A,B \rightarrow n \times n$
 $n = 134,484$





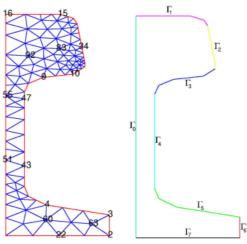
Scientific applications Industrial processes



- Optimal cooling of steel profiles
- Solve

$$A^T X + XA - XSX + Q = 0$$
,
dense $A \rightarrow n \times n$
 $n = 5,177$ for a mesh
width of $6.91 \cdot 10^{-3}$







Scientific applications Summary



- Dense linear algebra is at the bottom of the "food chain" for many scientific and engineering apps.
 - Fast acoustic scattering problems
 - Dielectric polarization of nanostructures
 - Magneto-hydrodynamics
 - Macro-economics



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Leveraging hw. concurrency Threads



Linear system

$$2x + 3y = 3$$

$$4x - 5y = 6$$

$$A X = B$$
, with dense A , B
→ $n \times n$:
≈ $2n^3/3 + 2n^3$ flops

Intel Xeon:

n	Time 1 core	Time 8 cores	Time 16-node cluster, 8-core per node, i.e., 192 cores
100	33.33 ms		-
1.000	0.33 s		
10 ⁴	333.33 s	41.62 s	
10 ⁵	> 92 h	> 11 h	> 28 m



Leveraging hw. concurrency Threads



2010 PFLOPS (10¹⁵ flops/sec.)

2010 JUGENE

- 10⁹ core level
 (PowerPC 450, 850MHz → 3.4 GFLOPS)
- 10¹ node level

(Quad-Core)

10⁵ cluster level

(73.728 nodes)



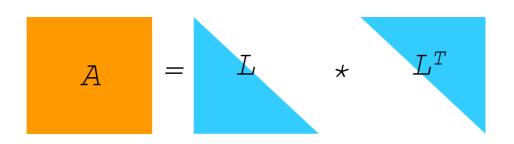
2020 EFLOPS (10¹⁸ flops/sec.)

- 10^{9.5} core level
- 10³ node level!
- 10^{5.5} cluster level



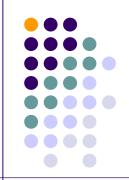
Leveraging hw. concurrency Cholesky factorization

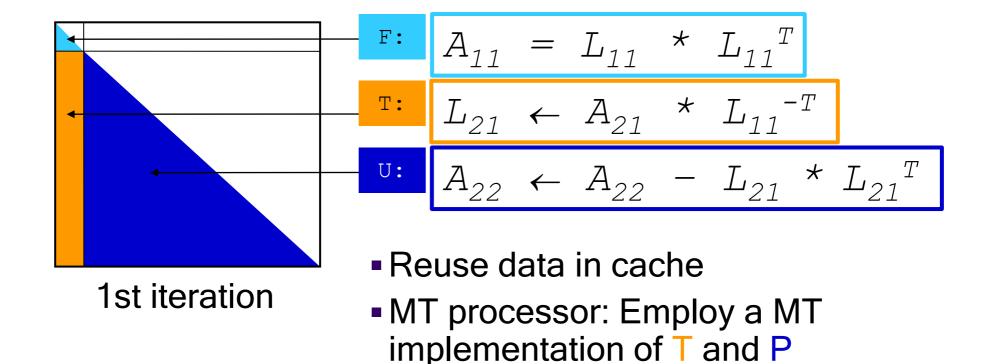




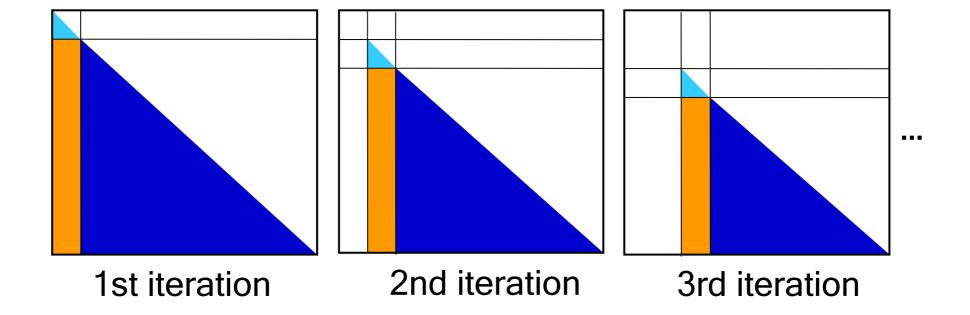
Key in the solution of s.p.d. linear systems



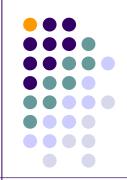






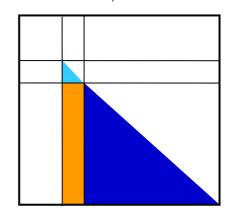




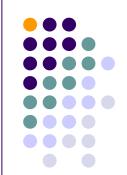


```
for (k=1; k<=n/b; k++) {

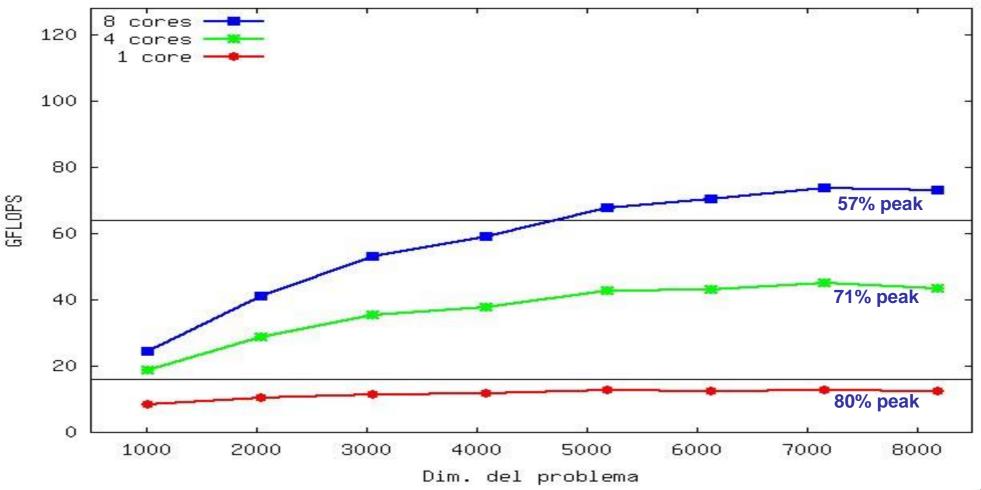
Chol(A[k,k]); // A<sub>kk</sub> = L<sub>kk</sub> * L<sub>kk</sub>
```







Factor de Cholesky en 2 Intel Xeon QuadCore (8 cores)





Leveraging hw. concurrency Algorithmic parallelism



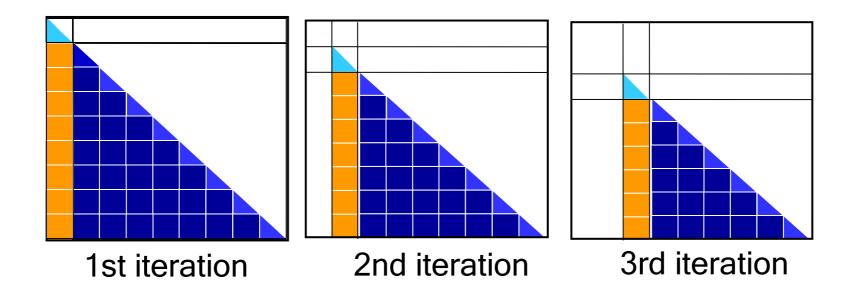
Why?

Excessive thread synchronization

Leveraging hw. concurrency Algorithmic parallelism



...but there is much more parallelism!!!

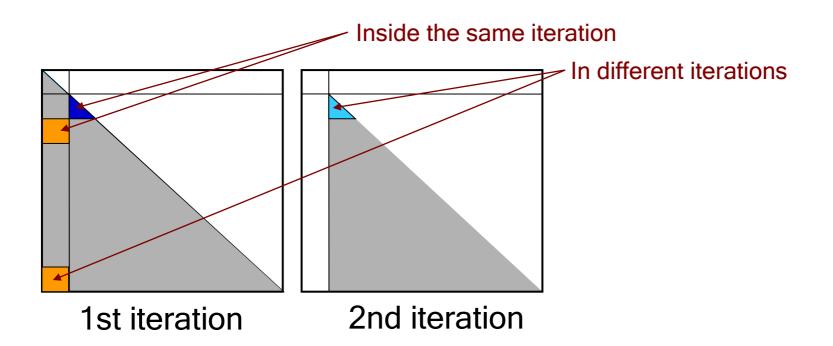




Leveraging hw. concurrency Algorithmic parallelism



...but there is much more parallelism!!!

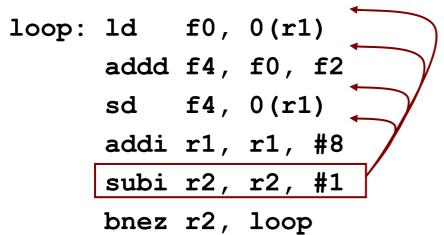


How can we leverage it?

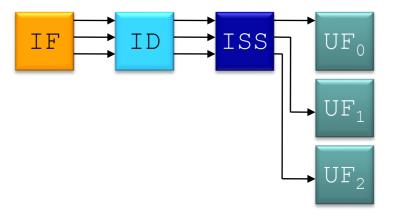




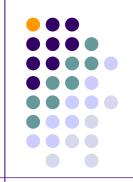
Scalar code



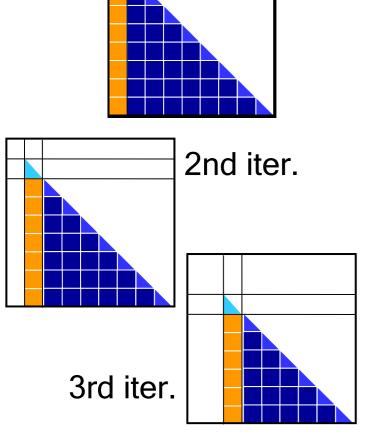
(Super)scalar processor







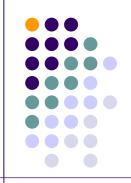
Something similar for (dense) linear algebra?



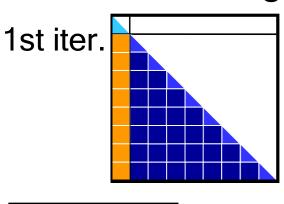
1st iter.

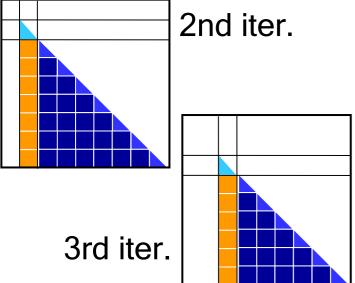
```
for (k=1; k<=n/b; k++) {
F: Chol(A[k,k]);
  for (i=k+1; i<=n/b; i++)
T: Trsm(A[k,k], A[i,k]);
  for (i=k+1; i<nb; i++) {
U1: Syrk(A[i,k],A[i,i]);
  for (j=k+1; j<=i; j++)
U2: Gemm(A[i,k], A[j,k], A[i,j]);
}</pre>
```





Something similar for (dense) linear algebra?





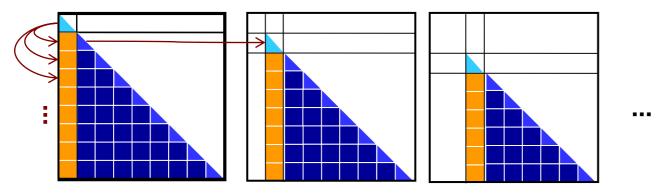
- Apply "scalar" techniques at the block level
- Software implementation
- Thread/Task-level parallelism
- Target the cores/GPUs of the platform



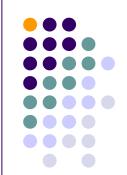


Read/written blocks determine dependencies, as in scalar case

Dependencies form a dependency DAG (task tree)

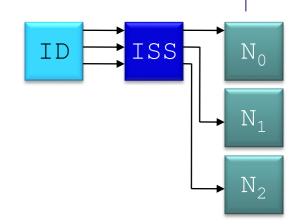




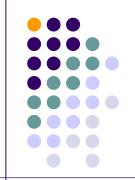


Runtime:

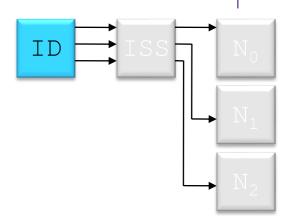
- Decode (ID): Generate the task tree with a "symbolic analysis" of the code at execution time
- Issue (ISS): Architectureaware execution of the tasks in the tree





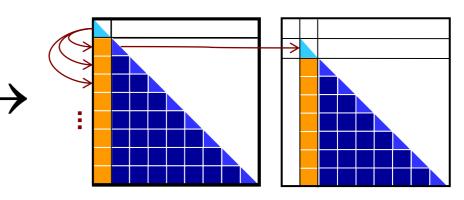


- Decode stage:
 - "Symbolic analysis" of the code

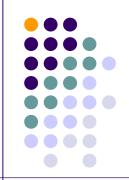


Blocked code:

Task tree:

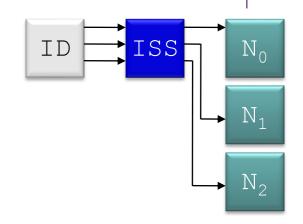


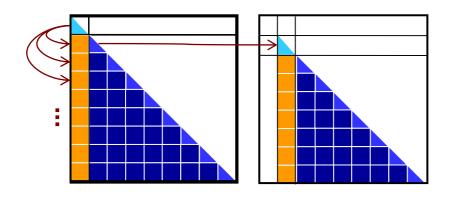




Issue stage:

- Temporal scheduling of tasks, attending to dependencies
- Mapping (spatial scheduling) of tasks to resources, aware of locality









Leveraging hw. concurrency Implementations



- SuperMatrix (UT@Austin and UJI)
 - Read/written blocks defined implicitly by the operations
 - Only valid for dense linear algebra operations encoded in libflame
- SMPSs (BSC) and GPUSs (BSC and UJI)
 - OpenMP-like languages
 #pragma css task inout(A[b*b])
 void Chol(double *A);
 - Applicable to task-parallel codes on different platforms: multi-core, multi-GPU, multi-accelerators, Grid,...



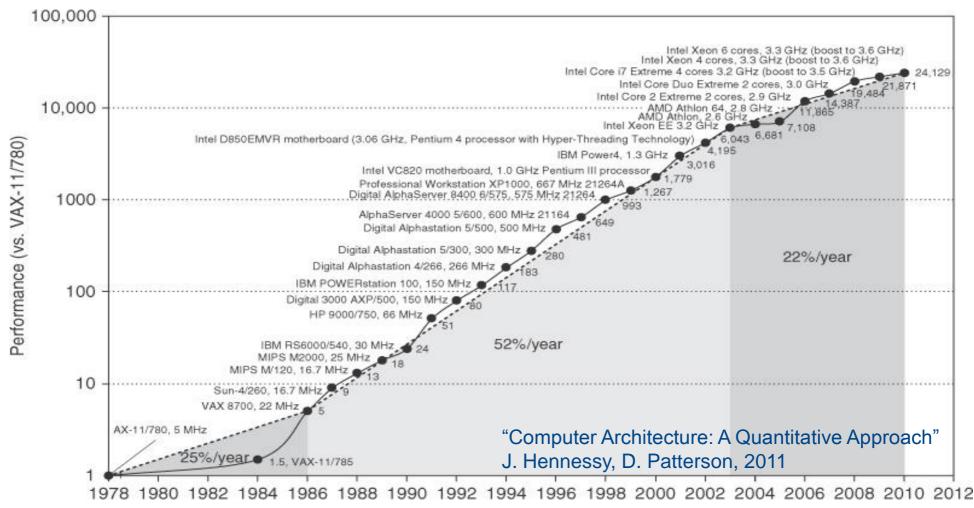
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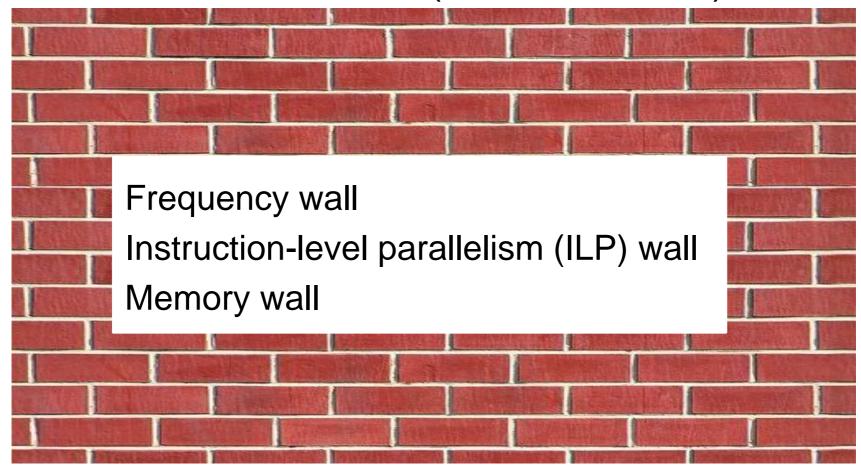








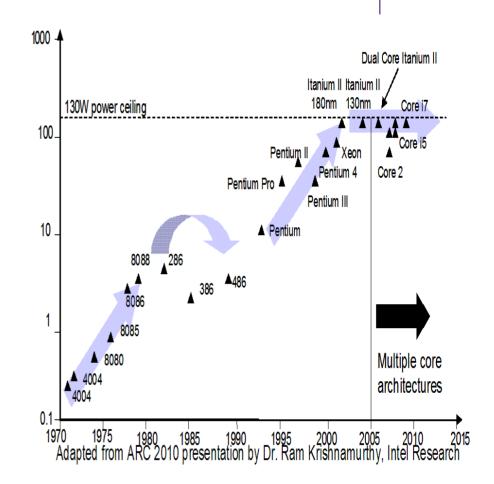
"The free lunch is over" (H. Sutter, 2005)







- Frequency wall
 - Power energy
 consumption proportional
 to f³ f²
 - Electricity = money
 - 1st Law of Thermodynamics: Energy cannot be created or destroyed, only converted
 - Cost of extracting heat
 - Heat reduces lifetime







Rank	Site	#Cores	MFLOPS/W	LINPACK (TFLOPS)	MW to EXAFLOPS?
Green/Top				(11 LOF 3)	LAAI LOFS!
1/29	IBM Rochester – BlueGene/Q, Power BQC 16C 1.60 GHz	32.768	2.026.48	339,83	493,47
7/114	BSC (Spain) – Bull B505, Xeon E5649 6C 2.53 GHz, NVIDIA 2090	5.544	1.266,26	103,20	789,73
32/1	RIKEN AICS K Computer– Spar64 VIIIfx (8-core)	705.024	830,18	10.510,00	1.204,60

NVIDIA GTX 480 (250 W) (=1/4 low power hair dryer) 2 million GTXs \approx 493,47 MW! or 500.000 hair dryers







Rank	Site	#Cores	MFLOPS/W	LINPACK (TFLOPS)	MW to EXAFLOPS?
Green/Top				(TFLOPS)	EXAFLOPS!
1/29	IBM Rochester – BlueGene/Q, Power BQC 16C 1.60 GHz	32.768	2.026.48	339,83	493,47
7/114	BSC (Spain) – Bull B505, Xeon E5649 6C 2.53 GHz, NVIDIA 2090	5.544	1.266,26	103,20	789,73
32/1	RIKEN AICS K Computer– Spar64 VIIIfx (8-core)	705.024	830,18	10.510,00	1.204,60



Most powerful reactor under construction in France Flamanville (EDF, 2017 for US \$9billion): 1,630 MWe



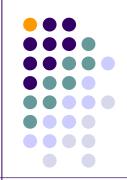
Cost of energy Setup



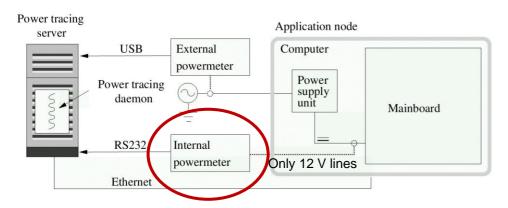
- Modeling power of task-parallel apps.
 - Two Intel Xeon E5504 @ 2.0 GHz (8 cores)
 - Experience: more stable
- Saving opportunities for task-parallel apps.
 - Two AMD Opteron 6128 cores @ 2.0 GHz (16 cores)
 - Experience: more flexible (DVFS at core level)



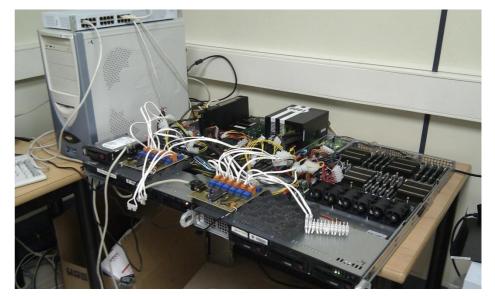
Cost of energy Setup



- DC powermeter with sampling freq. = 25 Hz
 - LEM HXS 20-NP transductors with PIC microcontroller
 - RS232 serial port



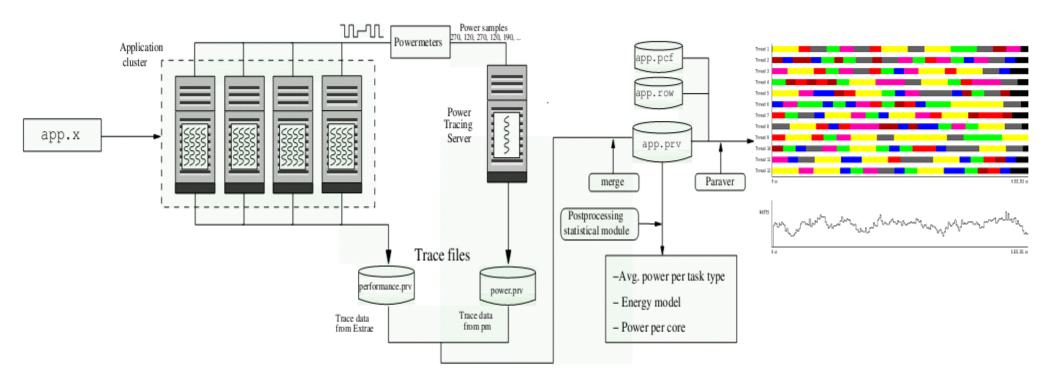






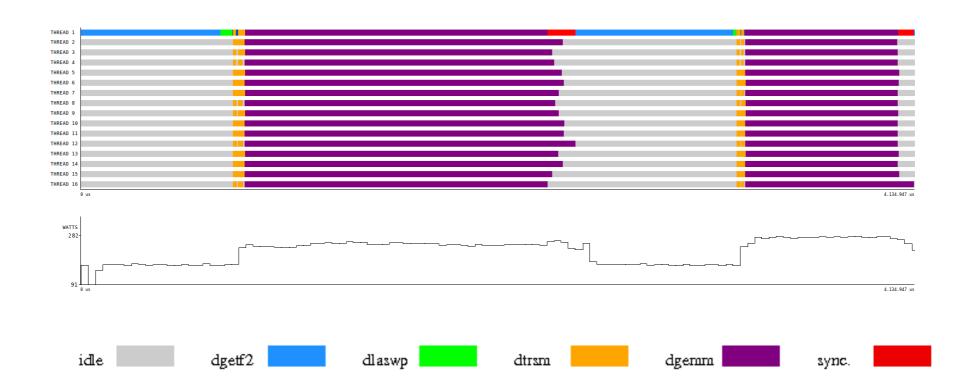
Cost of energy Setup





Cost of energy Setup



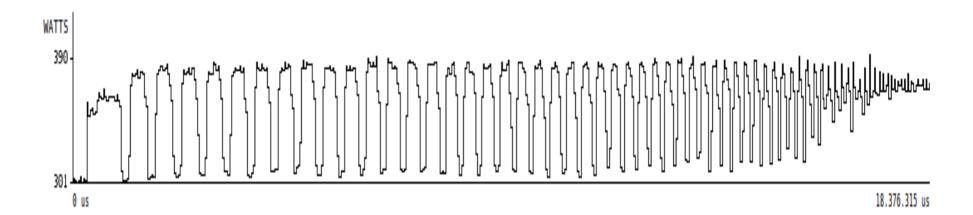




Cost of energy Power vs. energy

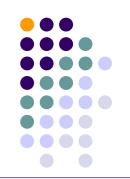


$$E = \int_0^T P \, dt$$

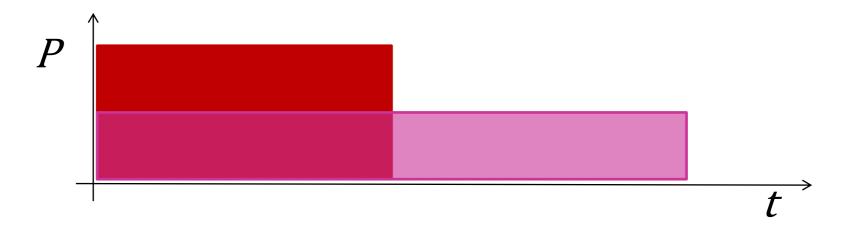




Cost of energy Power vs. energy

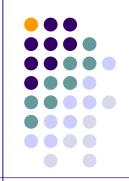


Which one is better, A or B?



$$E_A = P_A t_A$$
$$E_B = P_B t_B$$





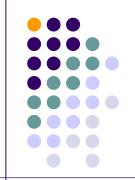
$$P = P^{(S)Y(stem)} + P^{C(PU)} = P^{Y} + P^{S(tatic)} + P^{D(ynamic)}$$

 P^{C} is power dissipated by CPU (socket): $P^{S} + P^{D}$ P^{Y} is power of remaining components (e.g., RAM)

Considerations:

- P^{Y} and P^{S} are constants (though P^{S} grows with temperature)
- Hot system
- Task-parallel routines
- Intel platform



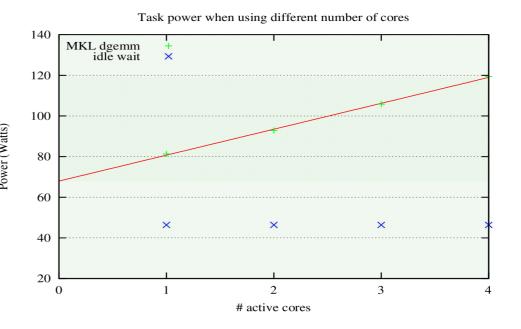


System power:

$$P = P^Y + P^S + P^D$$

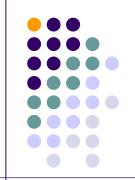
Estimated as *idle* power

Due to off-chip components:
e.g, RAM (only mainboard)



$$P^{Y} \approx P^{I} = 46.37 \text{ W}$$



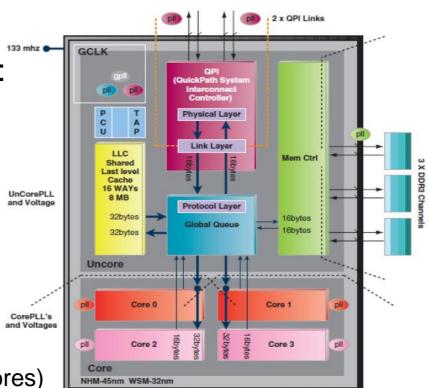


Static power:

$$P = P^Y + P^S + P^D$$

Also known as *Uncore* power (Intel):

- LLC
- Mem. controller
- Interconnect controller
- Power control logic
- etc.

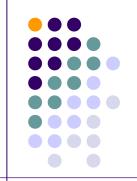


Intel Xeon 5500 (4 cores)

The Uncore: A Modular Approach to Feeding the High-performance Cores.

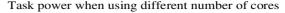
D. L. Hill et al. Intel Technology Journal, Vol. 14(3), 2010

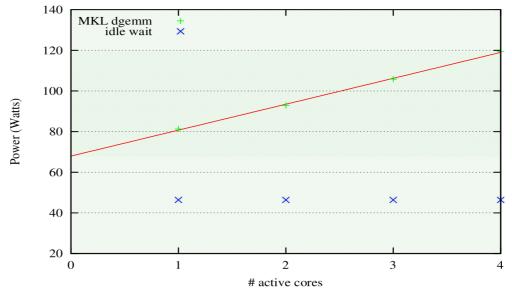




Static power:

$$P = P^Y + P^S + P^D$$

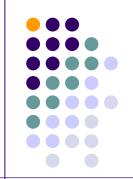




$$P_{dgemm}(c) = 67.97 + 12.75 c$$

 $P^{S} = 67.97 - 46.37 = 21.6 W$





Dynamic power:

$$P = P^Y + P^S + P^D$$

Also known as *Core* power (Intel):

- Execution units
- L1 and L2 cache
- Branch prediction logic
- etc.

UnCorePLL and Voltage

CorePLL's and Voltages

Pill Core 0

Core 2

Core 2

Core 2

Core 1

Core 3

Core 1

Core 3

Core 1

Core 3

Core 1

Core 3

Core 1

Core 1

Core 1

Core 3

Core 4

Core 4

Core 4

Core 4

Core 5

Core 5

Core 5

Core 6

Core 6

Core 7

Core 7

Core 7

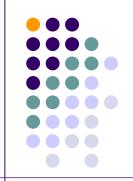
Core 8

Core 9

Co

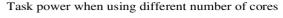
Intel Xeon 5500 (4 cores)

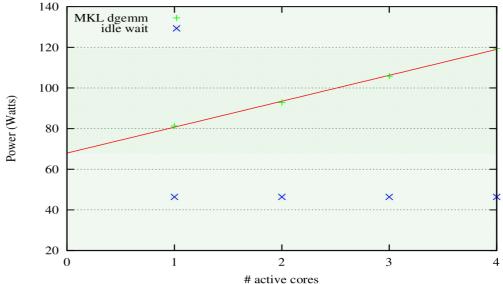




Dynamic power:

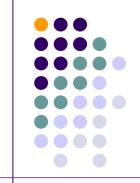
$$P = P^Y + P^S + P^D$$



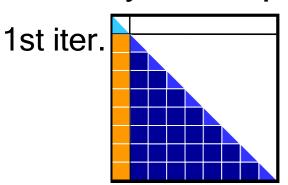


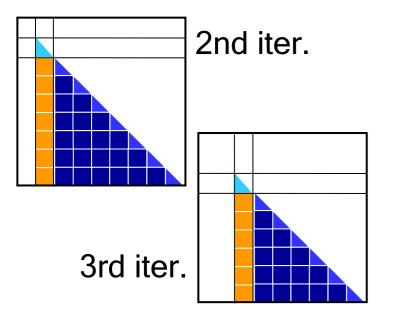
$$P_{dgemm}(c) = 67.97 + 12.75 c$$





Dynamic power of task-parallelCholesky

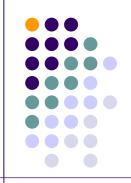




```
P = P^Y + P^S + P^D
```

```
for (k=1; k<=n/b; k++) {
F: Chol(A[k,k]);
  for (i=k+1; i<=n/b; i++)
T: Trsm(A[k,k], A[i,k]);
  for (i=k+1; i<nb; i++) {
    Syrk(A[i,k],A[i,i]);
    for (j=k+1; j<=i; j++)
    Gemm(A[i,k], A[j,k], A[i,j]);
  }
}</pre>
```





Dynamic power of task-parallelCholesky

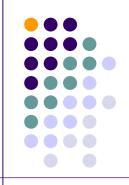
For a given kernel, execute repeatedly till power stabilizes:

$$P^{D}_{dgemm} = Pd_{gemm} - (P^{Y} + P^{S})$$

	1 kernel mapped to 1 core Block size, b				2 kernels mapped to 2 cores of different sockets			
					Block size, b			
Task	128	192	256	512	128	192	256	512
P_P^D (dpotrf)	10.26	10.35	10.45	11.28	9.05	9.09	9.28	10.44
P_T^D (dtrsm)	10.12	10.31	10.32	10.80	9.45	9.57	9.60	11.08
P_S^D (dsyrk)	11.22	11.47	11.67	12.60	10.42	10.63	10.82	11.80
P_G^D (dgemm)	11.98	12.54	12.72	13.30	10.90	12.16	11.28	11.96
P_B^D (busy)	7.62	7.62	7.62	7.62	7.62	7.62	7.62	7.62

Power increases linearly with #cores, from 1 to 4 mapped to a single socket When two sockets are used, linear function changes





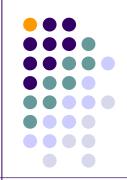
Power of task-parallel Cholesky

$$P_{chol} = P^{Y} + P^{S} + \sum_{i=1}^{r} \sum_{j=1}^{c} P^{D}_{i} N_{i,j}(t)$$

where

- r is #different types of tasks
- cis #cores
- P^{D}_{i} is the average dynamic power for task of type
- $N_{i,j}(t) = 1$ if thread j is executing a task of type i at time t; $N_{i,j}(t) = 0$ otherwise





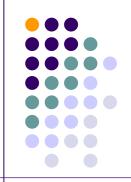
Energy of task-parallel Cholesky

$$E_{chol} = (P^{Y} + P^{S}) T + \sum_{i=1}^{r} \sum_{j=1}^{c} P^{D}_{i} \int_{t=0}^{T} N_{i,j}(t) dt$$
$$= (P^{Y} + P^{S}) T + \sum_{i=1}^{r} \sum_{j=1}^{c} P^{D}_{i} T_{i,j}$$

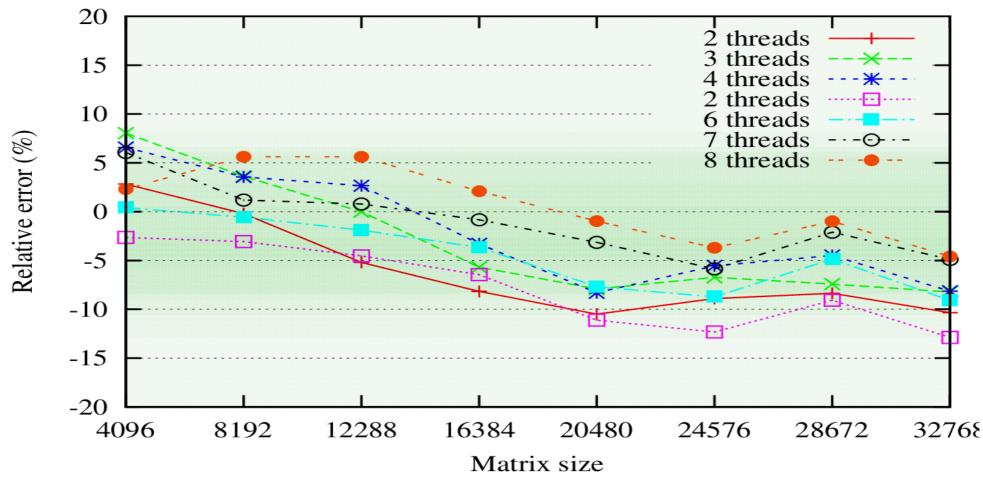
where

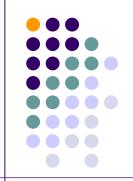
- T is the total execution time
- $T_{i,j}(t) = 1$ is the time that thread j has executed tasks of type i



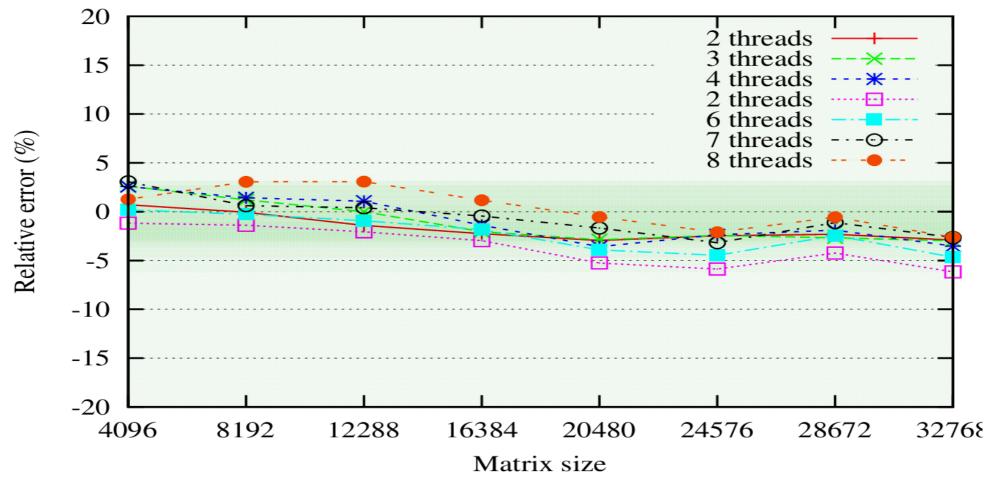


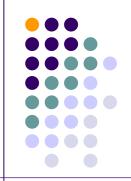
Error in dynamic energy consumption, b=128



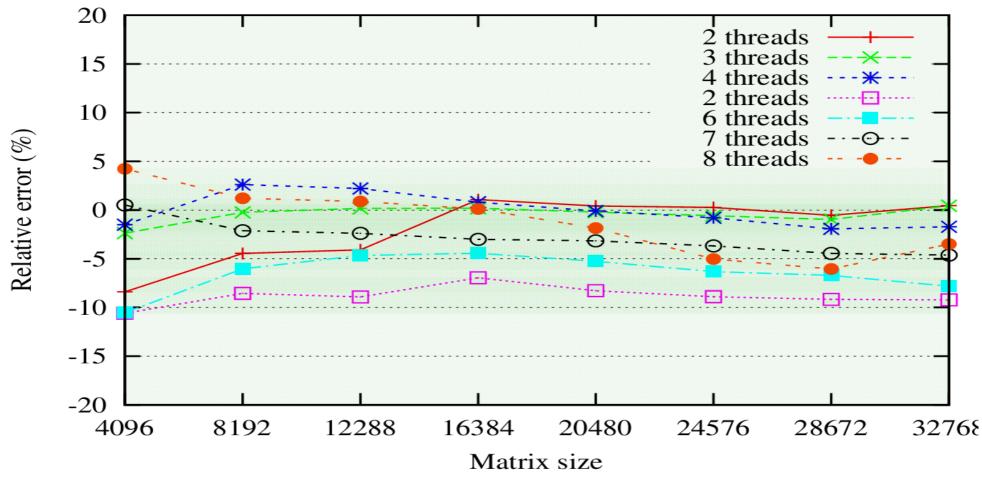


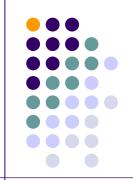
Error in total energy consumption, b=128



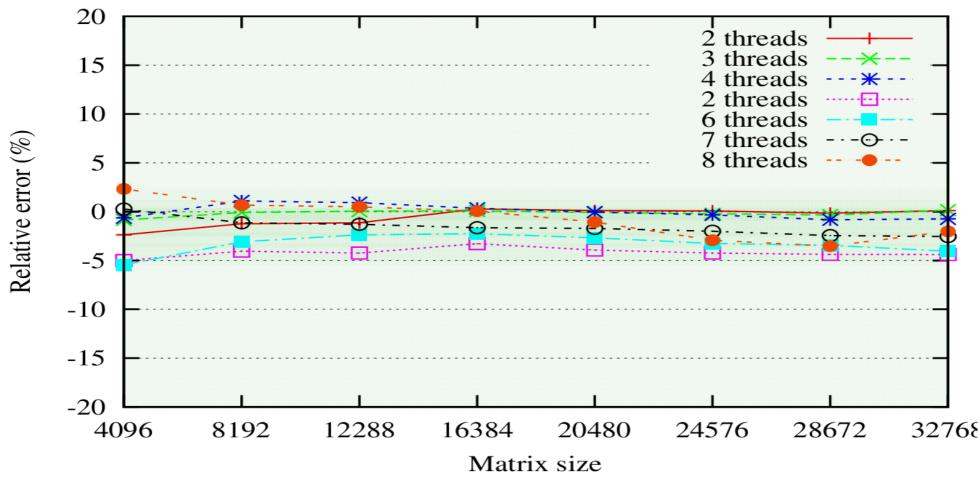


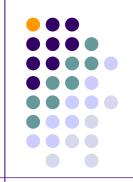
Error in dynamic energy consumption, b=256





Error in total energy consumption, b=256





 ACPI (Advanced Configuration and Power Interface): industry-standard interfaces enabling OS-directed configuration, power/thermal management of mobile/desktop/server platforms











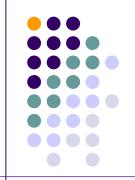
- Revision 5.0 (december 2011)
- In the processor: Power states (C-states) and performance states (P-states)





- Power states (C-states):
 - C0: normal execution (also a P-state)
 - Cx, x>0: no instructions being executed. As x grows, more savings but longer latency to reach C0
 - Stop clock signal
 - Flush and shutdown cache (L1 and L2 flushed to LLC)
 - Turn off cores





- Package power states (PC-states):
 - PC0, PC1, PC2,...

Uncore subsystem remains active and consumes power as long as there is any active core on the CPU

UncorePLL and Voltage

UncorePLL's and Voltages

| Core |

Intel Xeon 5500 (4 cores)

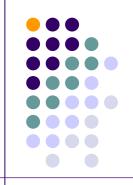




Intel Core i7 processor:

- Core C0 State
 - The normal operating state of a core where code is being executed
- Core C1/C1E State
 - The core halts; it processes cache coherence snoops
- Core C3 State
 - The core flushes the contents of its L1 instruction cache, L1 data cache, and L2 cache to the shared L3 cache, while maintaining its architectural state. All core clocks are stopped at this point. No snoops
- Core C6 State
 - Before entering core C6, the core will save its architectural state to a dedicated SRAM on chip. Once complete, a core will have its voltage reduced to zero volts



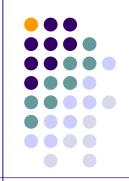


- Performance states (P-states):
 - P0: Highest performance and power
 - Pi, i > 0: As igrows, more savings but lower performance

P-state P_i	VCC_i	f_i	
P_0	1.23	2.00	
P_1	1.17	1.50	AMD platform
P_2	1.12	1.20	
P_3	1.09	1.00	
P_4	1.06	0.80	

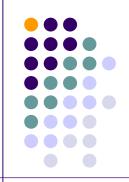
• $P = a V^2 f$, where a depends on the technology (but $E = \int_0^T P dt = a V^2$)





Leveraging DVFS: cpufreq



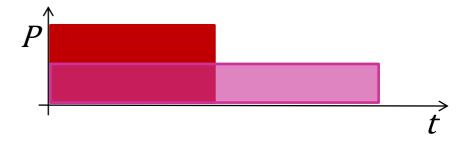


- Leveraging DVFS (transparent): Linux governors
 - Performance: Highest frequency/performance
 - Powersave: Lowest frequency/performance
 - Userspace: User's decision
 - Ondemand: If CPU utilization rises above the threshold value set in the up_threshold parameter, the ondemand governor increases the CPU frequency to scaling_max_freq. When CPU utilization falls below this threshold, the governor decreases the frequency in steps. Lowest performance, growing with workload
 - **Conservative**: If CPU utilization is above up_threshold, this governor will step up the frequency to the next highest frequency below or equal to scaling_max_freq. If CPU utilization is below down_threshold, this governor will step down the frequency to the next lowest frequency until it reaches scaling_min_freq

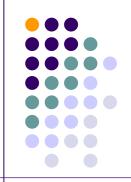




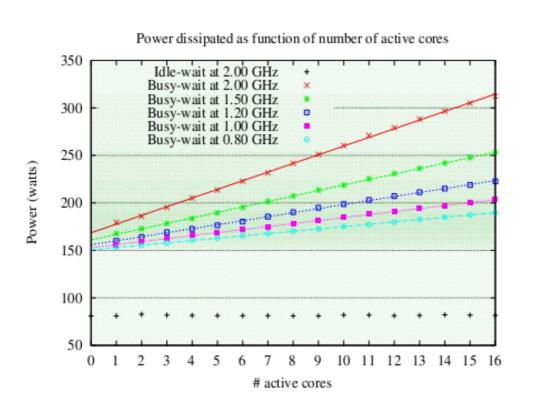
Which one is better, A or B?

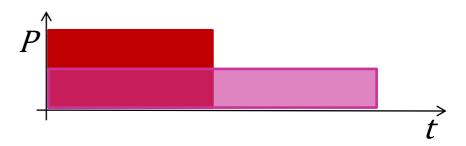




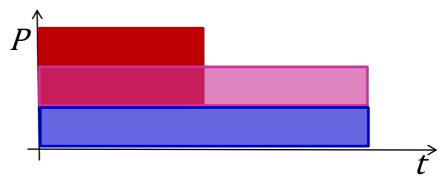


Which one is better, A or B?

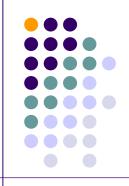




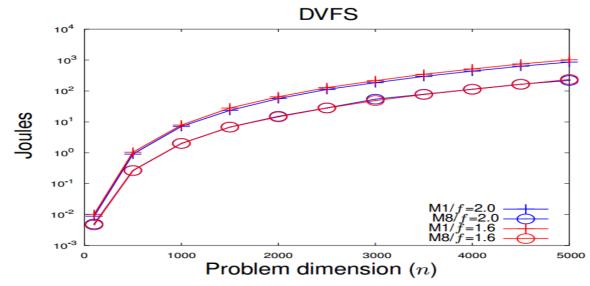
But consideralso $P^Y + P^S \simeq 50\%$ of power





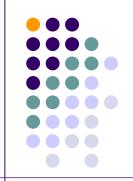


- To DVFS or not? General consensus:
 - No for compute-intensive apps.: reducing frequency increases execution time linearly



 Yes for memory-bounded apps. as cores are idle most of the time





 ...but, in many platforms, reducing frequency via DVFS also reduces memory bandwidth proportionally!

P-state P_i	VCC_i	f_i	α_i	β_i	ΔP_i^S	ΔP_i^D	$\Delta P_i^T(16)$	BW_i	ΔBW_i
P_0	1.23	2.00	168.59	9.12	_	_	_	4.43	_
P_1	1.17	1.50	161.10	5.77	-9.52	-32.14	-17.58	3.89	-12.19
P_2	1.12	1.20	155.90	4.23	-17.09	-50.25	-28.34	3.49	-21.21
P_3	1.09	1.00	152.94	3.15	-21.47	-60.73	-33.26	3.19	-27.99
P_4	1.06	0.80	150.61	2.44	-25.73	-70.30	-39.85	2.80	-36.79



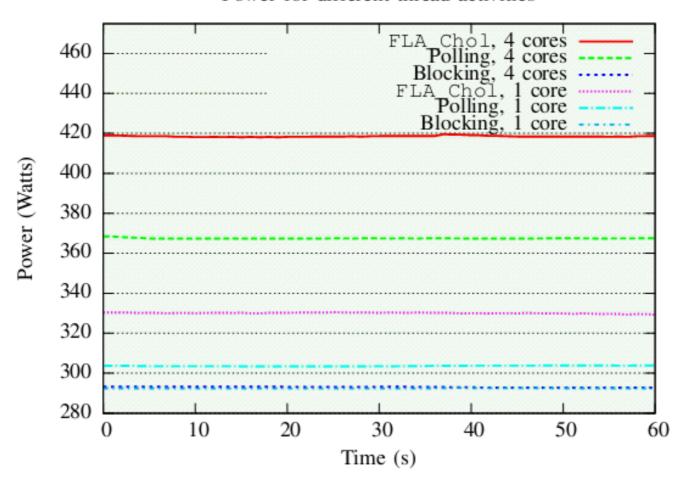


- Alternative strategies for compute-intensive apps.:
 - Idle-wait in multithreaded apps.
 - Idle-wait in hybrid CPU-GPU apps.
 - Idle-wait during communications in MPI apps.





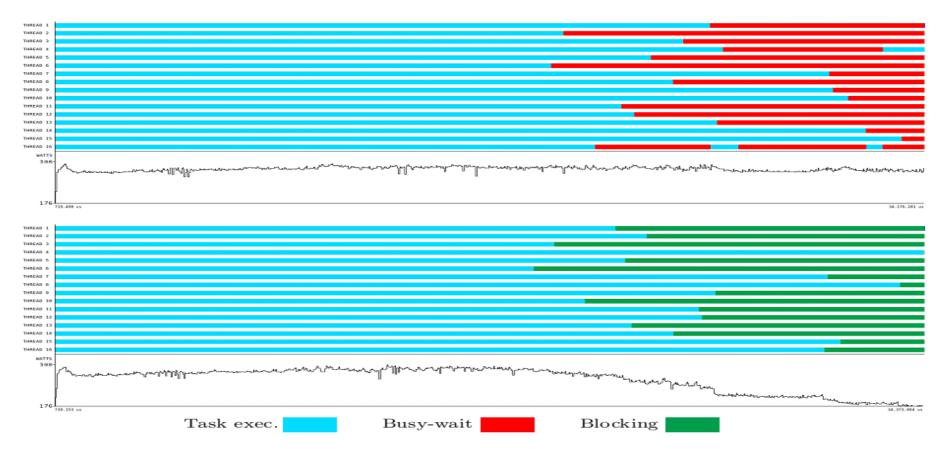
Power for different thread activities



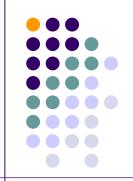




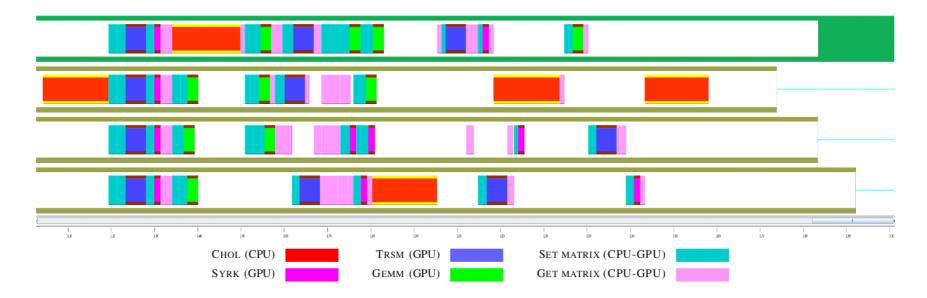
Idle-wait in multithreaded apps. (ILU preconditioner)







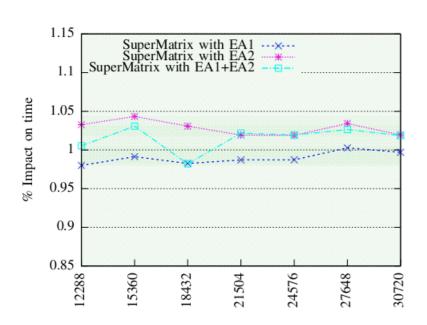
 Idle-wait in hybrid CPU-GPU apps. (multi-GPU Cholesky factorization via SuperMatrix runtime)



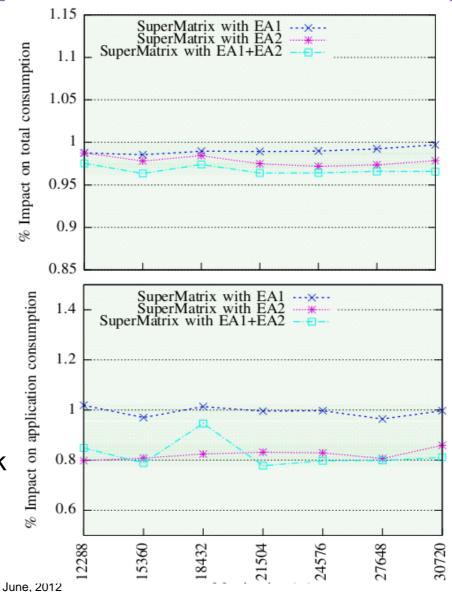
 Intel Xeon E5540 @ 2.83 GHz (4 cores) and NVIDIA Tesla S2050 (4 "Fermis")







EA1: no polling when there is no work EA2: no polling when work is in GPU





Performance and energy consumption Summary



- A battle to be won in the core arena
 - More concurrency
 - Heterogeneous designs
- A related battle to be won in the power arena
 - Do nothing, efficiently... (V. Pallipadi, A. Belay)
 - Don't forget the cost of uncore power

...but don't always believe the salesman!



